Inflationary Cosmology from Supergravity

来自超引力的暴胀宇宙学

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Abstract

摘要

The supergravity models for cosmological inflation and dark matter are introduced by assuming their (super)gravitational origin. Inflation is known to be sensitive to quantum gravity needed for its ultraviolet completion. Supergravity

本文通过假设宇宙学暴涨与暗物质的超引力模型源于 (超) 引力，对这些模型进行介绍。众所周知，暴涨对量子引力敏感，而量子引力是暴涨紫外完备性的必需。超引力

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is considered as the meeting point of high-energy physics and cosmology, as well as the bridge between classical and quantum gravities. String theory is a mathematically consistent theory of quantum gravity that requires space-time supersymmetry. Modified supergravity theories are introduced as the extensions of the Starobinsky model of inflation based on modified gravity. Physical applications of the supergravity models include viable single-field and multi-field inflation, high-scale spontaneous supersymmetry breaking, primordial black hole production, and dark matter genesis. Supermassive gravitino particles and primordial black holes are the viable candidates for the dark matter originating from supergravity. High-precision measurements of the cosmic microwave background radiation by satellite missions and detection of the gravitational waves, induced by primordial black hole formation, by the future space-based gravitational interferometers can be viewed as the possible observational tests of the cosmological models based on supergravity theory.

被认为是高能物理与宇宙学的交汇点，也是经典引力与量子引力之间的桥梁。弦理论是数学上自洽的量子引力理论，它要求时空超对称性。本文介绍的修正超引力理论是基于修正引力的斯塔罗宾斯基暴涨模型的推广。这类超引力模型的物理应用包括可行的单场与多场暴涨、高尺度自发超对称破缺、原初黑洞产生以及暗物质起源。超重引力微子粒子和原初黑洞都是源于超引力的暗物质的可行候选者。卫星任务对宇宙微波背景辐射的高精度测量，以及未来空间引力干涉仪对原初黑洞形成诱导引力波的探测，都可看作对这类基于超引力理论的宇宙学模型的可能观测检验。

Keywords

关键词

Inflation - Supergravity - Dark matter - Primordial black holes

暴胀 - 超引力 - 暗物质 - 原初黑洞

Introduction

引言

The oldest signal from the past to the present is given by the cosmic microwave background (CMB) emerged when electromagnetic radiation decoupled from matter about 380,000 years after the birth of the universe. It leads to the observational wall against any electromagnetic probe because photons did not propagate freely before that time due to Thompson scattering. Extracting observables from the earlier universe may only be possible from gravitational waves (GW) or neutrino sources.

从过去留存至今最古老的信号是宇宙微波背景 (CMB)，它诞生于宇宙诞生约 38 万年后，当时电磁辐射与物质退耦。宇宙微波背景形成了我们利用电磁探测探索更早宇宙的观测边界，因为在此之前，由于汤姆逊散射，光子无法自由传播。只有通过引力波 (GW) 或中微子源，才有可能提取早期宇宙的可观测信息。

Information about the CMB spectrum, its anisotropies, and fluctuations is available due to the satellite missions in the past (COBE, WMAP, Planck). Theoretical explanation of the CMB observations is possible by assuming a very short era of cosmological inflation in the early universe. The cosmological paradigm of inflation assumes an accelerated (quasi-de Sitter) expansion of the universe with a "graceful exit," which greatly amplified microscopic fluctuations that became seeds of the large-scale structure of the universe we observe today. From the viewpoint of particle physics, inflation was the most powerful particle accelerator in Nature, well beyond the standard model (SM). The CMB gives us the great (though small) window into very high-energy physics beyond the SM. The inflationary paradigm also solves the old (flatness, horizon, initial conditions) problems of the standard (Friedmann) cosmology.

过去的卫星任务 (COBE、WMAP、普朗克) 已经为我们获取了 CMB 的能谱、各向异性与涨落相关信息。通过假设早期宇宙曾经历一段极短的暴胀时期，可以对 CMB 观测结果给出理论解释。暴胀宇宙学范式假设宇宙经历了带有” 优雅退出” 的加速 (类德西特) 膨胀，这一过程极大放大了微观涨落，这些涨落最终成为我们如今观测到的宇宙大尺度结构的种子。从粒子物理的角度来看，暴胀是自然界中最强大的粒子加速器，能量远超出标准模型 (SM) 的范围。CMB 为我们提供了一个探索超出标准模型的超高能物理的极佳窗口 (尽管窗口很小)。暴胀范式还解决了标准 (弗里德曼) 宇宙学中存在的老问题 (平直性问题、视界问题、初始条件问题)。

The simplest mechanisms of inflation employ the canonical scalar field called inflaton whose scalar potential defines a single-field inflation model. However, the physical nature of inflation and the origin of its scalar potential are still unknown. Matching CMB observations leaves many viable models of inflation, so that their discrimination requires adding other principles. One of such principles, namely, the possible gravitational origin of inflation, is extended to supergravity and dark matter (DM) in this chapter.

最简单的暴胀机制采用了名为暴胀子的正则标量场，其标量势定义了单场暴胀模型。但暴胀的物理本质及其标量势的起源至今仍不明确。符合 CMB 观测的可行暴胀模型仍有很多，因此要区分这些模型需要引入其他原理。本文将其中一种原理——即暴胀可能起源于引力——扩展到超引力和暗物质 (DM) 的研究中。

In section ”Starobinsky Inflation,” we review the Starobinsky model of inflation having the gravitational origin. More general inflation models motivated by the primordial black hole (PBH) formation are defined in section ”Generalized Models of Inflation and PBH.” The supergravity setup is given in section ”Supergravity and Inflation.” A minimal embedding of inflation to supergravity is given in section ”Minimal Embedding of Inflation to Supergravity.” Section ”Starobinsky-Type Supergravity and PBH Production” is devoted to PBH production in modified supergravity models of inflation. The GW induced by the PBH production are studied in section ”Induced Gravitational Waves.” Section ”Adding Matter and Spontaneous SUSY Breaking” is devoted to spontaneous supersymmetry (SUSY) breaking with PBH production. Gravitino production in the Polonyi-Starobinsky supergravity and the massive gravitino DM are considered in section ”Gravitino DM Genesis.” Our conclusion is section ”Conclusion.”

在” 斯塔罗宾斯基暴胀” 一节中，我们回顾了起源于引力的斯塔罗宾斯基暴胀模型。在” 广义暴胀模型与原初黑洞” 一节中，我们介绍了由原初黑洞 (PBH) 形成动机驱动的更广义的暴胀模型。” 超引力与暴胀” 一节给出了超引力框架。” 暴胀到超引力的最小嵌入” 一节给出了暴胀嵌入超引力的最小方案。” 斯塔罗宾斯基型超引力与原初黑洞产生” 一节专门研究修正超引力暴胀模型中的 PBH 产生问题。我们在” 诱导引力波” 一节中研究了 PBH 产生诱导的引力波 (GW)。” 加入物质与超对称自发破缺” 一节专门研究伴随 PBH 产生的超对称 (SUSY) 自发破缺。我们在” 引力微子暗物质起源” 一节中研究了博洛尼-斯塔罗宾斯基超引力中的引力微子产生，以及有质量引力微子暗物质。我们的结论放在” 结论” 一节。

Starobinsky Inflation

斯塔罗宾斯基暴胀

The gravitational origin of inflation implies that only gravitational interactions can be used for its description. This leads to modified gravity and the Starobinsky inflation model [1]. In this section, the Starobinsky model of inflation is reviewed, without following historical developments.

暴胀的引力起源意味着仅能依靠引力相互作用对其描述，这催生了修正引力与斯塔罗宾斯基暴胀模型 [1]。本节我们将回顾斯塔罗宾斯基暴胀模型，不遵循其发展历史。

A spatially (flat) homogeneous and isotropic (1+3)-dimensional universe at large scales (beyond 100Mpc) is described by the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric

大尺度 (超出 100Mpc 范围) 下空间均匀各向同性的 (1+3) 维平直宇宙，由空间平直弗里德曼-勒梅特-罗伯逊-沃尔克 (FLRW) 度规描述

$$ds^2_{\mathrm{FLRW}} = -dt^2 + a^2 \left(d\vec{x}\right)^2, \tag{1}$$

where the function $a(t)$ is called the cosmic scale factor. Cosmological inflation is referred to the very early (after 10^{-36} s and before 10^{-32} s) accelerating universe with

其中函数 $a(t)$ 被称为宇宙标度因子。宇宙暴胀指发生在极早期 (10^{-36} s 之后、10^{-32} s 之前) 的加速膨胀宇宙，满足

$$\dot{a}(t) > 0 \tag{2}$$

and a "graceful exit" quickly after inflation. The Hubble radius (causal distance) H^{-1}/a , where $H = \dot{a}/a$ is the Hubble function, was decreasing during inflation:

并在暴胀结束后快速“优雅退出”。哈勃半径 (因果距离) H^{-1}/a ，其中 $H = \dot{a}/a$ 为哈勃函数，在暴胀过程中不断减小:

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0. \tag{3}$$

The single-field mechanism of inflation uses the scalar field called inflaton, whose potential energy drives inflation. The corresponding "quintessence" action reads

单场暴胀机制使用被称为暴胀子的标量场，由其势能驱动暴胀，对应的“精质”作用量为

$$S_{\text{single}}\left[g_{\mu\nu}, \varphi\right] = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4x\sqrt{-g}R - \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right],$$

(4)

where we have introduced the reduced Planck mass $M_{\mathrm{Pl}} = 1/\sqrt{8\pi G_{\mathrm{N}}} \approx 2.4\times\ 10^{18}\mathrm{GeV}$.

这里我们引入约化普朗克质量 $M_{\mathrm{Pl}} = 1/\sqrt{8\pi G_{\mathrm{N}}} \approx 2.4\times\ 10^{18}\mathrm{GeV}$ 。

The Starobinsky model [1] is defined by the modified gravity action

斯塔罗宾斯基模型 [1] 由修正引力作用量定义

$$S_{\text{Star.}} = \frac{M_{\text{Pl}}^2}{2} \int d^4x\sqrt{-g}\left(R + \frac{1}{6m^2}R^2\right), \tag{5}$$

where we have introduced the inflaton mass parameter m . The action (5) reduces to the Einstein-Hilbert gravitational action in the low curvature regime. In the high curvature regime relevant to inflation, the action (5) reduces to the no-scale R^2 gravity action with the positive dimensionless coupling constant $M_{\text{Pl}}^2/\left(12m^2\right)$. It guarantees the absence of ghosts and tachyons in the model.

我们引入了暴胀子质量参数 m 。作用量 (5) 在低曲率区域退化为爱因斯坦-希尔伯特引力作用量。在与暴胀相关的高曲率区域，作用量 (5) 退化为无标度 R^2 引力作用量，具有正无量纲耦合常数 $M_{\text{Pl}}^2/\left(12m^2\right)$ ，这保证了模型中不存在鬼场和快子。

The R^2 -gravity equations of motion in the FLRW universe have a quasi-de Sitter attractor solution with the "graceful exit," whose leading term during slow-roll inflation is

FLRW 宇宙中 R^2 引力的运动方程存在一个具有“优雅退出”性质的类德西特吸引子解，其在慢滚暴胀阶段的主导项为

$$H(t) \approx \left(\frac{m}{6}\right)^2 (t_{\text{end}} - t), \tag{6}$$

for $m(t_{\text{end}} - t) \gg 0$. This solution spontaneously breaks the scale invariance of the R^2 -gravity and, hence, implies the existence of the associated Nambu-Goldstone boson (inflaton) called scalaron. The easiest way to extract the scalaron φ from the higher-derivative gravity theory (5) is via the known (classical) equivalence between the modified $F(R)$ gravity theories and the scalar-tensor gravity theories [2]. The equivalence is realized via the Legendre-Weyl transform:

对应 $m(t_{\text{end}} - t) \gg 0$ 。该解自发破缺了 R^2 引力的标度不变性，因此预示了存在名为标量子的 associated 南部-戈德斯通玻色子 (即暴胀子)。从高阶导数引力理论 (5) 中提取标量子 φ 最简单的方法，是利用修正 $F(R)$ 引力理论与标量-张量引力理论之间已知的 (经典) 等价性 [2]，该等价性通过勒让德-外尔变换实现:

$$\varphi = \sqrt{\frac{3}{2}}M_{\text{Pl}} \ln F'(\chi) \quad \text{and} \quad g_{\mu\nu} \to \frac{2}{M_{\text{Pl}}^2}F'(\chi)g_{\mu\nu},\ \chi = R, \tag{7}$$

leading to the inflaton scalar potential

得到暴胀子标量势

$$V(\chi) = \left(\frac{M_{\text{Pl}}^2}{2}\right)\frac{\chi F'(\chi) - F(\chi)}{F'(\chi)^2} \tag{8}$$

and the canonical kinetic term for φ minimally interacting with gravity, as in Eq. (4).

以及如式 (4) 所示的，与引力最小耦合的 φ 正则动能项。

The inverse transformation reads

逆变换为

$$R=\left[\frac{\sqrt{6}}{M_{\rm Pl}}V'(\varphi)+\frac{4V(\varphi)}{M_{\rm Pl}^2}\right]\exp\left(\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right), \tag{9}$$

$$F=\left[\frac{\sqrt{6}}{M_{\rm Pl}}V'(\varphi)+\frac{2V(\varphi)}{M_{\rm Pl}^2}\right]\exp\left(2\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right), \tag{10}$$

defining the function $F(R)$ in the parametric form for a given inflaton scalar potential $V(\varphi)$. The flatness of the inflaton potential (needed for slow roll during a sufficient duration of inflation) amounts to the smallness of the first term against the second one in the square brackets of Eqs. (9) and (10). For instance, ignoring the first term leads to $F\propto R^2$.

它针对给定的暴胀子标量势 $V(\varphi)$ ，以参数形式定义了函数 $F(R)$ 。暴胀持续足够时间所需的慢滚条件要求暴胀子势足够平坦，对应式 (9) 和 (10) 方括号中第一项远小于第二项。例如，忽略第一项后得到 $F\propto R^2$ 。

The exact inflaton potential $V(\varphi)$ of the Starobinsky model (5) according to Eqs. (7) and (8) is given by

根据式 (7) 和 (8)，斯塔罗宾斯基模型 (5) 的精确暴胀子势 $V(\varphi)$ 为

$$V(\varphi)=\frac{3}{4}M_{\rm Pl}^2m^2\left[1-\exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\right]^2. \tag{11}$$

The potential (11) is a sum of the induced cosmological constant and the exponentially small corrections for large values of φ , where the potential has a plateau of positive height.

当 φ 取值很大时，势函数 (11) 是诱导宇宙学常数与指数小修正的和，此时势函数具有一个正高度的平台。

The parameter m of the Starobinsky model is fixed by the observed CMB amplitude as

Starobinsky 模型的参数 m 由观测到的 CMB 振幅确定为

$$m\approx 3\cdot 10^{13}\mathrm{GeV}\ \text{or}\ \frac{m}{M_{\rm Pl}}\approx 1.3\cdot 10^{-5}. \tag{12}$$

The ultraviolet (UV) cutoff of the quantized $\left(R+\alpha R^2\right)$ gravity is given by $M_{\rm Pl}$, as is clear from expanding the nonrenormalizable scalar potential (11) in powers of φ . This feature leads to expected protection of the Starobinsky inflation on the scale $H_{\rm inf.}\sim 10^{14}\mathrm{GeV}$ against quantum gravity corrections on the scale $M_{\rm Pl}$.

量子化 $\left(R+\alpha R^2\right)$ 引力的紫外 (UV) 截断由 $M_{\rm Pl}$ 给出，这一点从将不可重整化标量势 (11) 按 φ 的幂次展开后就很清楚。该性质使得 Starobinsky 暴涨在 $H_{\rm inf.}\sim 10^{14}\mathrm{GeV}$ 能标上，可以免受量子引力在 $M_{\rm Pl}$ 能标的修正影响。

The duration of slow-roll inflation is measured by e-folds running backward with time:

慢滚暴涨的持续时间由逆着时间计算的 e 折数衡量:

$$N = -\ln\left(a/a_{\text{end}}\right) \approx \frac{1}{M_{\text{Pl}}^2}\int_{\varphi_{\text{end}}}^{\varphi_*} \frac{V}{V'} d\varphi, \tag{13}$$

where φ_* is the inflaton value at the horizon crossing and φ_{end} is the inflaton value at the end of inflation when one of the slow-roll parameters,

其中 φ_* 是视界穿越时的暴胀子取值，φ_{end} 是暴涨结束时的暴胀子取值，此时某个慢滚参数

$$\varepsilon_V(\varphi) = \frac{M_{\text{Pl}}^2}{2}\left(\frac{V'}{V}\right)^2 \text{ and } \eta_V(\varphi) = M_{\text{Pl}}^2\left|\frac{V''}{V}\right|, \tag{14}$$

is no longer small, being close to one.

不再很小，接近 1。

Metric perturbations on the FLRW background are given by

FLRW 背景下的度规扰动为

$$g_{ij}(x) = a^2(t)\left[(1-2\mathcal{R})\delta_{ij} + h_{ij}\right], \tag{15}$$

where $\mathcal{R}(x)$ describes scalar perturbations and $h_{ij}(x)$ describes tensor perturbations or primordial GW. Linearizing the action (4) on the FLRW background with respect to scalar perturbations yields the Mukhanov-Sasaki equation [3,4]

其中 $\mathcal{R}(x)$ 描述标量扰动，$h_{ij}(x)$ 描述张量扰动即原初引力波。将作用量 (4) 在 FLRW 背景下对标量扰动线性化，得到穆哈诺夫-佐佐木方程 [3,4]

$$\mathcal{R}''_{\vec{k}} + 2\frac{z'}{z}\mathcal{R}'_{\vec{k}} + k^2\mathcal{R}_{\vec{k}} = 0, \tag{16}$$

where $\mathcal{R}_{\vec{k}}$ is the 3D Fourier transform of $\mathcal{R}(x)$, $z = a\dot{\varphi}/H, k^2 = \vec{k}^2$, and the primes denote the derivatives with respect to conformal time τ defined by $ad\tau = dt$. Changing the variables as $u_{\vec{k}} = z\mathcal{R}_{\vec{k}}$ yields the harmonic oscillator equation

其中 $\mathcal{R}_{\vec{k}}$ 是 $\mathcal{R}(x)$, $z = a\dot{\varphi}/H, k^2 = \vec{k}^2$ 的三维傅里叶变换，撇号表示对共形时间 τ 求导，共形时间 τ 由 $ad\tau = dt$ 定义。按 $u_{\vec{k}} = z\mathcal{R}_{\vec{k}}$ 做变量替换后得到谐振子方程

$$u''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right)u_{\vec{k}} = 0, \tag{17}$$

with the time-dependent frequency.

其频率随时间变化。

The power spectrum of scalar perturbations is given by

标量扰动的功率谱为

$$P_s(k) = \frac{k^3}{2\pi^2}\left|\mathcal{R}_{\vec{k}}\right|^2, \tag{18}$$

while its tilt n_s is defined by

而它的倾斜量 n_s 定义为

$$n_s(k) - 1 = \frac{d \ln P_s}{d \ln k}. \tag{19}$$

As regards CMB, the tilt n_s is measured at the horizon crossing with $k^* = 0.05\text{Mpc}^{-1}$. Similar definitions apply to the power spectrum $P_g(k)$ of tensor perturbations, leading to the tensor-to-scalar ratio $r = P_g/P_s$.

对于 CMB，倾斜量 n_s 是在视界穿越时以 $k^* = 0.05\text{Mpc}^{-1}$ 测量得到的。张量扰动的功率谱 $P_g(k)$ 也有类似定义，由此得到张标比 $r = P_g/P_s$ 。

The Starobinsky model (5) is known to be the excellent model of inflation, in very good agreement with CMB measurements. The tilt of scalar perturbations is given by $n_s \approx 1 + 2\eta_V - 6\varepsilon_V \approx 0.9649 \pm 0.0042$ (with 68%CL), while the tensor-to-scalar ratio is given by $r \approx 16\varepsilon_V < 0.036$ (with 95%CL), according to the 2021 data [5,6]. The Starobinsky model gives $r \approx 12/N^2 \approx 0.004$ and $n_s \approx 1 - 2/N$, with the best fit at $N = 55$.

Starobinsky 模型 (5) 是公认极佳的暴涨模型，与 CMB 测量结果吻合得非常好。根据 2021 年的数据 [5,6]，标量扰动倾斜量为 $n_s \approx 1 + 2\eta_V - 6\varepsilon_V \approx 0.9649 \pm 0.0042$ (对应 68%CL)，张标比为 $r \approx 16\varepsilon_V < 0.036$ (对应 95%CL)。Starobinsky 模型给出 $r \approx 12/N^2 \approx 0.004$ 和 $n_s \approx 1 - 2/N$ ，最佳拟合点为 $N = 55$ 。

After rewriting the scalar potential (11) to the form of a mass term by the field redefinition,

通过场重新定义将标量势 (11) 改写为质量项形式后，

$$\sqrt{\frac{3}{2}} M_{\text{Pl}} \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\text{Pl}}\right)\right] = \phi, \tag{20}$$

one gets a noncanonical kinetic term of the field ϕ , with the pre-factor having a singularity at $\phi_{\text{cr.}} = \sqrt{3/2}M_{\text{Pl}}$ and the critical exponent $\sqrt{\frac{2}{3}}$ defining the universality class of the Starobinsky-like inflation models. This approach is known as the pole inflation.

我们得到场 ϕ 的非正则动能项，其前置因子在 $\phi_{\text{cr.}} = \sqrt{3/2}M_{\text{Pl}}$ 处存在奇点，临界指数 $\sqrt{\frac{2}{3}}$ 定义了类 Starobinsky 暴涨模型的普适类。这套方法被称为极点暴涨。

The CMB data gives a small window into high-energy physics of inflation, while no strong observational constraints are available for the scales beyond the inflationary scale or a few orders of magnitude smaller.

CMB 数据为暴胀的高能物理研究提供了一个小窗口，而对于高于暴胀能标，或是比暴胀能标低数个数量级的能标，目前尚无强有力的观测约束。

Generalized Models of Inflation and PBH

暴胀与原初黑洞的广义模型

Primordial density fluctuations in the early universe (during or after inflation) may be responsible for PBH seeds when their amplitude is larger by the factor of 10^7 compared to the CMB amplitude. The very idea of PBH was proposed a long time ago by Zel'dovich and Novikov [7] and also by Hawking [8]. PBH may survive in the current universe and thus are the candidates for (non-particle) DM [9]. The PBH idea attracted a lot of attention in connection to GW [10-12].

当早期宇宙 (暴胀期间或暴胀后) 原初密度涨落的振幅比 CMB 振幅大 10^7 倍时，其涨落可能形成原初黑洞种子。原初黑洞的概念最早由泽尔多维奇、诺维科夫 [7] 和霍金 [8] 提出。原初黑洞可以存留至今，因此是 (非粒子) 暗物质的候选者 [9]。原初黑洞因引力波 (GW) 受到大量关注 [10-12]。

The PBH-DM is the viable alternative to the explanations of DM in particle physics, such as DM being composed of the weakly interacting massive particles (WIMP) like neutralino or axions [13]. Supergravity offers yet another possibility that DM is composed of the supergravitationally interacting massive particles (SGIMP) such as massive gravitinos [14]. Given part of DM in the form of PBH, one should search for DM signals in cosmological data rather than in direct detection on colliders or via indirect detection in astro-particle physics. Another reason for PBH studies is due to observational progress in lensing, cosmic rays, GW detection, and CMB radiation measurements; see, for example, Ref. [10] for a review of observational constraints on PBH. PBH may also offer a solution to several astrophysical puzzles, for example, the existence of supermassive black holes.

原初黑洞形式的暗物质是粒子物理暗物质解释的可行替代方案, 粒子物理中暗物质通常由弱相互作用大质量粒子 (WIMP，比如中性微子) 或轴子构成 [13]。超引力还提出了另一种可能性: 暗物质由超引力相互作用大质量粒子 (SGIMP，比如大质量引力微子) 构成 [14]。如果部分暗物质以原初黑洞的形式存在, 那么人们应该在宇宙学数据中而非对撞机直接探测或天体粒子物理间接探测中寻找暗物质信号。研究原初黑洞的另一个动机来自引力透镜、宇宙线、引力波探测和 CMB 辐射测量的观测进展；关于原初黑洞观测限制的综述，例如可见文献 [10]。原初黑洞还可以解决多个天体物理谜题，例如超大质量黑洞的起源问题。

There are many possible mechanisms that may catalyze the formation of PBH in the early universe, for instance, (i) gravitational instabilities induced by scalar fields [15] in single-field or multi-field inflation, (ii) bubble collisions from first-order phase transitions [16-18], and (iii) formation of critical topological defects such as cosmic strings [19] and domain walls [20,21]. The leading scenario at present is given by (i).

早期宇宙中有多种可能机制催化原初黑洞形成，例如:(i) 单场或多场暴胀中由标量场诱导的引力不稳定性 [15]，(ii) 一级相变的泡碰撞 [16-18]，(iii) 宇宙弦 [19]、畴壁 [20,21] 等临界拓扑缺陷的形成。目前主流的场景是 (i)。

PBH can also be considered as a probe of very high-energy physics and quantum gravity, "even if they never formed" [22]. Several phenomenological scenarios were proposed for PBH formation and, especially, for PBH generation after inflation in the early universe, under the assumption that PBH significantly contribute to DM; see, for example, Refs. [23-27] and the references therein. The whole PBH-DM appears to allow only two limited windows for the PBH masses, either around 10^{-15} or around 10^{-12} of the solar mass. These masses are much less the masses of the black holes whose mergers resulted in the GW observed by the LIGO detector [28].

原初黑洞甚至可以作为高能物理和量子引力的探针，“即使它们从未形成”[22]。在原初黑洞对暗物质有显著贡献的假设下，人们已经提出了多种关于原初黑洞形成，尤其是早期宇宙暴胀后原初黑洞产生的唯象学场景；例如可见文献 [23-27] 及其中的参考文献。整体来看，原初黑洞构成全部暗物质的情况仅允许两个有限的原初黑洞质量窗口: 质量约为 10^{-15} 倍太阳质量，或约为 10^{-12} 倍太阳质量。这些质量远小于 LIGO 探测器观测到的并合产生引力波的黑洞质量 [28]。

Since the current absence of observed non-Gaussianities and isocurvature perturbations in the CMB data [29], the single-field models were distinguished in the literature about inflation and PBH formation, also due to their simplicity. The Starobinsky model is one of the most favored models of single-field inflation, but its sharp predictions for the tilts n_s and r may be ruled out by future CMB measurements. The Starobinsky model also does not allow PBH production.

目前 CMB 数据中未观测到非高斯性和等曲率扰动 [29]，且单场模型更为简单，因此在关于暴胀和原初黑洞形成的研究中单场模型得到了重视。斯塔罗宾斯基模型是最受青睐的单场暴胀模型之一，但它对倾斜参数 n_s 和 r 的明确预测可能会被未来的 CMB 观测排除。斯塔罗宾斯基模型也不支持原初黑洞产生。

Getting arbitrary values of r is possible by using the alpha-attractor generalizations [30] of the Starobinsky model. For instance, replacing the critical exponent $\sqrt{\frac{2}{3}}$ in the Starobinsky potential (11) by $\sqrt{\frac{2}{3\alpha}}$ with arbitrary $\alpha > 0$ gives the new (called E-type) inflationary models having the potential

利用斯塔罗宾斯基模型的 α 吸引子推广可以得到 r 的任意值。例如，将斯塔罗宾斯基势 (11) 中的临界指数 $\sqrt{\frac{2}{3}}$ 替换为带有任意 $\alpha > 0$ 的 $\sqrt{\frac{2}{3\alpha}}$ ，就得到了一类新的 (称为 E 型) 暴胀模型，其势为

$$V_\alpha(\varphi) = V_0\left[1 - \exp\left(-\sqrt{\frac{2}{3\alpha}}\varphi/M_{\rm Pl}\right)\right]^2. \tag{21}$$

The key feature of the alpha-attractors is the value of the tensor-to-scalar ratio (on CMB scales)

α 吸引子的核心特征是 (CMB 尺度上的) 张量标量比为

$$r_\alpha \approx \frac{12\alpha}{N_e^2} \tag{22}$$

while they have $n_s = 1 - 2/N_e$ (on CMB scales) as in the Starobinsky model.

同时 (CMB 尺度上的) $n_s = 1 - 2/N_e$ 和斯塔罗宾斯基模型一致。

The alpha-attractors can be generalized to the (T-type) inflaton potentials [30] α 吸引 or 可以推广为 (T 型) 暴胀子势 [30]

$$V_{\tilde{\alpha},f}(\varphi) = f^2\left(\tanh\frac{\kappa\varphi}{\sqrt{6\alpha}}\right) \tag{23}$$

with a monotonically increasing (during slow roll) function f , and $\kappa = M_{\rm Pl}^{-1}$. In these models, slow-roll inflation occurs for large positive values of the canonical inflation field φ with the approximate scalar potential $(\kappa = 1)$

其中 f 是慢滚过程中单调递增的函数，且满足 $\kappa = M_{\rm Pl}^{-1}$ 。在这类模型中，慢滚暴胀发生在正则暴胀场 φ 取大正值的区域，近似标势为 $(\kappa = 1)$

$$V(\varphi) = f_\infty^2 - 4f_\infty f'_\infty e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + O\left(e^{-2\sqrt{\frac{2}{3\alpha}}\varphi}\right), \tag{24}$$

where we have introduced the parameters $f_\infty = f|_{\varphi\to\infty}$ and $f'|_\infty = \partial_\varphi f\big|_{\varphi\to\infty}$. The constant in front of the second term in Eq. (24) can be adjusted at will by a constant shift of the inflaton field, so that the potential (24) can be simplified to

此处我们引入了参数 $f_\infty = f|_{\varphi\to\infty}$ 和 $f'|_\infty = \partial_\varphi f\big|_{\varphi\to\infty}$ 。式 (24) 第二项前的常数可通过 inflaton 场的常数平移任意调整，因此势函数 (24) 可简化为

$$V(\varphi) = V_0\left(1 - 2e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\right) + O\left(e^{-2\sqrt{\frac{2}{3\alpha}}\varphi}\right), \tag{25}$$

thus establishing the asymptotic equivalence to the E-type alpha-attractors on CMB scales.

由此证明其在 CMB 尺度上与 E 型 α 吸引子渐近等价。

The enhancement of the power spectrum of scalar perturbations (needed for PBH formation) can be achieved by engineering a near-inflection point in the inflaton potential [31,32]. Details of the PBH production in single-field inflation are dependent upon a choice of the inflaton potential and require fine-tuning of the parameters. For instance, it can be realized via Taylor expansion of the function f , when keeping the first three terms as [33],

标量扰动功率谱的增强 (PBH 形成所需条件) 可通过在 inflaton 势中构造一个近拐点实现 [31,32]。单场通胀中 PBH 产生的细节依赖于 inflaton 势的选取，且需要对参数进行精细调节。例如，保留前三项时，可通过对函数 f 做泰勒展开实现，如文献 [33] 所示，

$$V_{\rm T}(\phi) = V_0\left[1 + c_1\tanh\frac{\kappa\phi}{\sqrt{6\tilde{\alpha}}} + c_2\tanh^2\frac{\kappa\phi}{\sqrt{6\tilde{\alpha}}} + c_3\tanh^3\frac{\kappa\phi}{\sqrt{6\tilde{\alpha}}}\right]^2, \tag{26}$$

after fine-tuning of the parameters V_0 and c_i for $i = 1, 2, 3$. The same goal can be achieved by a similar deformation of the E-type scalar potential (21) as [34]

在针对 $i = 1, 2, 3$ 精细调节参数 V_0 和 c_i 之后。相同目标也可通过对 E 型标量势 (21) 做类似变形实现，如文献 [34] 所示

$$V_E(\varphi) = V_0 \left[1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi/M_{Pl}} + \beta e^{-2\sqrt{\frac{2}{3\alpha}}\varphi/M_{Pl}} - \gamma e^{-3\sqrt{\frac{2}{3\alpha}}\varphi/M_{Pl}} \right]^2 \tag{27}$$

with the tuned parameters V_0, β, γ . The generated PBH masses appear in the mass window of $10^{17} \nabla \cdot 10^{20}$ g [33, 34] .

参数 V_0, β, γ 经过调节后。产生的 PBH 质量出现在 $10^{17} \nabla \cdot 10^{20}$ g [33, 34] 的质量窗口内。

There are no fundamental reasons for the absence of non-Gaussianities and isocurvature perturbations; they just have to be below observational limits. For instance, PBH production may be a generic feature of two-field inflation with a sharp turn of inflationary trajectory [35]. The required growth of primordial fluctuations can be achieved by tachyonic instabilities of scalars, similar to the waterfall phase of hybrid inflation [36].

非高斯性和等曲率扰动不存在并没有根本原因；它们只是必须低于观测限制。例如，PBH 产生可能是具有尖锐转向的通胀轨迹的双场通胀的普遍特征 [35]。原初涨落所需的增长可通过标量的快子不稳定性实现，类似于混合通胀的瀑布阶段 [36]。

A multi-field action for describing inflation is given by

描述通胀的多场作用量为

$$S_{\text{multi}}\left[g_{\mu\nu}, \phi_a\right] = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} R$$

$$- \int d^4x \sqrt{-g} \left[\frac{1}{2} G^{ab}(\phi) g^{\mu\nu} \partial_\mu \phi_a \partial_\nu \phi_b + V(\phi) \right] \tag{28}$$

in terms of several (real) scalar fields $\phi_a, a = 1, 2, \ldots, n$. The scalar kinetic terms in Eq. (28) have the form of the nonlinear sigma-model (NLSM) [37] with the metric G^{ab} in the field space.

用多个 (实) 标量场 $\phi_a, a = 1, 2, \ldots, n$ 表示。式 (28) 中标量动能项具有非线性 σ 模型 (NLSM)[37] 的形式，场空间中带有度规 G^{ab} 。

Supergravity and Inflation

超引力与暴胀

Multi-field models of inflation and PBH formation increase physical degrees of freedom and possible interactions, which reduces their predictive power. It is, therefore, important to impose some fundamental symmetry principles to restrict physics beyond the SM of elementary particles. Supersymmetry (SUSY) is the fundamental symmetry unifying elementary particles of different spin into irreducible multiplets, restricting their interactions and independent coupling constants. In the context of gravity, one needs local SUSY, i.e., supergravity.

多场暴胀与原初黑洞 (PBH) 形成模型增加了物理自由度与可能的相互作用，这降低了它们的预言能力。因此，引入基本对称性原理来限制粒子物理标准模型 (SM) 之外的物理十分重要。超对称性 (SUSY) 是一种基本对称性，它能将不同自旋的基本粒子统一为不可约多重态，限制粒子间的相互作用与独立耦合常数。在引力范畴内，我们需要局域超对称性，也就是超引力。

Supergravity is also a good framework to study inflation and theoretical origin of PBH at the more fundamental level than general relativity because local SUSY transformations imply general coordinate transformations. Supergravity is also a bridge from classical gravity to quantum gravity when the latter is given by string theory because string theory requires SUSY for its consistency.

超引力也是比广义相对论更基础的研究框架，适合用于研究暴胀和原初黑洞的理论起源，因为局域超对称变换蕴含着一般坐标变换。若量子引力由弦论描述，超引力还是连接经典引力与量子引力的桥梁，因为弦论的自洽性要求超对称存在。

Despite the absence of experimental confirmation of SUSY on TeV-scales at the Large Hadron Collider (LHC), SUSY remains one of the leading candidates for new physics beyond SM of elementary particles for the scales beyond 100 TeV. SUSY has to be spontaneously or softly broken at the collider energies. The scale of SUSY breaking is unknown.

尽管大型强子对撞机 (LHC) 尚未在 TeV 能标得到超对称的实验证实，对于 100 TeV 以上的能标，超对称仍然是粒子物理标准模型之外新物理的核心候选之一。超对称在对撞机能标下必然是自发破缺或软破缺的，而超对称破缺的能标目前尚不清楚。

We confine ourselves to $N = 1$ supergravity in four space-time dimensions because it is chiral that is necessary for particle phenomenology and CP violation. SUSY and supergravity have many attractive theoretical features such as the following :

我们将讨论限定在四维时空的 $N = 1$ 超引力，因为它是手征的，这是粒子唯象学和 CP 破坏所要求的性质。超对称和超引力有诸多出色的理论性质，列举如下:

- SUSY unifies bosons and fermions.

  - 超对称统一玻色子与费米子。

- Supergravity includes general relativity.

  - 超引力包含广义相对论。

- SUSY grand unified theories (super-GUT) lead to a perfect unification of electroweak and strong interactions.

  - 超对称大统一理论 (超 GUT) 可以完美统一电弱相互作用与强相互作用。

- the spectrum of matter-coupled supergravities with spontaneously broken SUSY has the natural DM candidate given by the lightest SUSY particle (LSP) provided that R-parity is conserved.

- 若 R 宇称守恒，自发破缺超对称的物质耦合超引力能谱中，最轻超对称粒子 (LSP) 是天然的暗物质候选者。

- SUSY leads to the cancellation of the quadratic UV divergences in Feynman graphs and protection of chiral (F-type) actions in quantum perturbation theory

- 超对称可以消除费曼图中的二次紫外发散，在量子微扰论中保护手征 (F 型) 作用量。

- Supergravity is necessary to consistent coupling of spin-3/2 particles to gravity.

- 超引力是自旋 3/2 粒子与引力自洽耦合的必要条件。

- Supergravity arises as the low-energy effective action of superstrings and M-theory.

- 超引力是超弦和 M 理论的低能有效作用量。

However, the high-scale SUSY implies that it cannot stabilize the fundamental scales (the hierarchy problem), such as the electroweak scale and the GUT scale. In summary, SUSY and supergravity are healthy and are not ruled out by observations despite the absence of any signs of their presence by the LHC and in the sky so far.

但高标度超对称意味着它无法解决基本能标 (电弱标度和大统一标度) 的稳定性问题，即层级问题。总而言之，尽管到目前为止 LHC 和天体观测都没有发现超对称和超引力存在的任何迹象，它们的理论基础依然成立，并未被观测排除。

SUSY requires superpartners for each SM particle and equal numbers of bosonic and fermionic degrees of freedom, as well as supersymmeteric actions. To match observations and connect to the SM, SUSY has to be (spontaneously) broken. The formal technology of local supersymmetry is based on (i) the superconformal tensor calculus [38] and (ii) the curved superspace [39]. Both approaches are equivalent but technically involved. The $N = 1$ superspace technology is geometrical and offers manifest SUSY of supersymmetric actions. Being applied to inflation, it means that the bosonic actions (4) and (5) have to be locally supersymmetrized. It is worth mentioning that the modified gravity action (5) includes the higher derivatives so that its supersymmetrization goes beyond the textbooks. In addition, inflation has a positive energy (or a positive height of the potential) and thus breaks SUSY. Therefore, there must be a Goldstone spin-1/2 fermion called goldstino that is associated with spontaneous SUSY breaking during inflation.

超对称要求每个标准模型粒子都存在对应的超对称伙伴，要求玻色子和费米子自由度数量相等，同时要求作用量具有超对称性。为了符合观测结果并连接标准模型，超对称必须 (自发) 破缺。局域超对称的形式化方法基于两大体系:(i) 超共形张量微积分 [38]，(ii) 弯曲超空间 [39]。两种方法等价，但技术上都较为复杂。$N = 1$ 超空间方法是几何化的，可以让超对称作用量的超对称性明显可见。将该方法应用于暴胀，意味着公式 (4) 和 (5) 的玻色作用量必须局域超对称化。值得一提的是，修改引力作用量 (5) 包含高阶导数，因此它的超对称化超出了现有教材的范畴。此外，暴胀具有正能量 (即势具有正高度)，因此会破缺超对称。因此，暴胀过程中必然存在一个与自发超对称破缺相关的 Goldstone 自旋 1/2 费米子，称为戈德斯迪诺。

Both inflaton and goldstino have to belong to (irreducible) supermultiplets or superfields. Usually, both superfields are chosen to be chiral with the maximal spin 1/2 [40, 41] , which requires complexification of the inflaton and the need of two chiral superfields having four real scalars leading to multi-field inflation.

暴胀子和戈德斯迪诺都必须属于 (不可约) 超多重态或超场。通常二者都取手征超场，最大自旋为 1/2 [40, 41] ，这要求暴胀子复化，因此需要两个手征超场，对应四个实标量，最终形成多场暴胀。

Slow-roll inflation in supergravity is usually realized by engineering the scalar potential V in terms of a Kähler potential K and a superpotential W of the chiral superfields $\mathbf{\Phi}^i$. The standard Lagrangian for the chiral superfields coupled to supergravity is given by a sum of the D-term and F-term as follows ($M_{\rm Pl} = 1$) :

超引力中的慢滚暴胀通常通过构造标量势 V 实现，标量势由手征超场 $\mathbf{\Phi}^i$ 的凯勒势 K 和超势 W 给出。耦合超引力的手征超场的标准拉格朗日量由 D 项和 F 项相加得到，形式如下 ($M_{\rm Pl} = 1$) :

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left[\frac{3}{8}\left(\overline{\mathcal{D}}^2 - 8\mathcal{R}\right)e^{-K\left(\mathbf{\Phi}^i,\overline{\mathbf{\Phi}}^i\right)/3} + W\left(\mathbf{\Phi}^i\right)\right] + \text{ h.c.} , \tag{29}$$

where $\mathcal{E}$ is the chiral density superfield, $\mathcal{R}$ is the chiral curvature superfield, $\mathcal{D}_\alpha$ and $\overline{\mathcal{D}}_{\dot\alpha}$ are the superspace covariant derivatives, $\mathcal{D}^2 \equiv \mathcal{D}^\alpha\mathcal{D}_\alpha$, and $\overline{\mathcal{D}}^2 \equiv \overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}^{\dot\alpha}$; see Ref. [39] for the standard notation of supergravity in curved superspace. A non-holomorphic Kähler potential $K\left(\Phi^i,\overline{\Phi}^i\right)$ and a holomorphic superpotential $W\left(\Phi^i\right)$ define the model and uniquely determine its scalar sector in the form (28).

其中 $\mathcal{E}$ 是手征密度超场，$\mathcal{R}$ 是手征曲率超场，$\mathcal{D}_\alpha$ 和 $\overline{\mathcal{D}}_{\dot\alpha}$ 是超空间协变导数，$\mathcal{D}^2 \equiv \mathcal{D}^\alpha\mathcal{D}_\alpha$ ，以及 $\overline{\mathcal{D}}^2 \equiv \overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}^{\dot\alpha}$ ；弯曲超空间中超引力的标准符号参见文献 [39]。非全纯凯勒势 $K\left(\Phi^i,\overline{\Phi}^i\right)$ 和全纯超势 $W\left(\Phi^i\right)$ 定义了该模型，并以式 (28) 的形式唯一确定了其标量 sector。

A chiral superfield in terms of its field components reads

手征超场用其场分量表示为

$$\mathbf{\Phi}(x,\Theta) = \Phi(x) + \sqrt{2}\Theta\chi(x) + \Theta^2 F(x) . \tag{30}$$

After eliminating the auxiliary fields (F) via their algebraic equations of motion and going to Einstein frame by a Weyl transform of metric, the bosonic part of the Lagrangian (29) is given by

通过辅助场 (F) 的代数运动方程消去辅助场，并通过外尔变换将度量变换到爱因斯坦规范后，拉格朗日量 (29) 的玻色子部分为

$$e^{-1}\mathcal{L} = \frac{1}{2}R - K_{i\bar{j}}\partial_m\Phi^i\partial^m\overline{\Phi}^j - e^K\left(K^{i\bar{j}}D_iWD_{\bar{j}}\bar{W} - 3|W|^2\right), \tag{31}$$

where we have used the same notation for the superfields and their leading field components, together with

其中我们对超场及其领头场分量采用了相同的记号，且有

$$K_{i\bar{j}} \equiv \frac{\partial^2 K}{\partial \Phi^i \partial \overline{\Phi}^j},\ K^{i\bar{j}} \equiv K^{-1}_{i\bar{j}},\ D_i W \equiv \frac{\partial W}{\partial \Phi^i} + W\frac{\partial K}{\partial \Phi^i}. \tag{32}$$

Therefore, we have

因此我们得到

$$V = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right). \tag{33}$$

The Θ -expansion of the chiral superfields $\mathcal{E}$ and $\mathcal{R}$ of supergravity in terms of their field components is given by

超引力中手征超场 $\mathcal{E}$ 和 $\mathcal{R}$ 按场分量展开的 Θ 展开式为

$$2\mathcal{E} = e\left[1 + i\Theta\sigma^m\overline{\psi}_m + \Theta^2\left(6\bar{X} - \overline{\psi}_m\overline{\sigma}^{mn}\overline{\psi}_n\right)\right], \tag{34}$$

$$\mathcal{R} = X + \Theta\left(-\frac{1}{6}\sigma^m\overline{\sigma}^n\psi_{mn} - i\sigma^m\overline{\psi}_m X - \frac{i}{6}\psi_m b^m\right) +$$

$$+\Theta^2\left(-\frac{1}{12}R - \frac{i}{6}\overline{\psi}^m\overline{\sigma}^n\psi_{mn} - 4X\bar{X} - \frac{1}{18}b_m b^m + \frac{i}{6}\nabla_m b^m +\right.$$

$$\left.+\frac{1}{2}\overline{\psi}_m\overline{\psi}^m X + \frac{1}{12}\psi_m\sigma^m\overline{\psi}_n b^n - \frac{1}{48}\varepsilon^{abcd}\left(\overline{\psi}_a\overline{\sigma}_b\psi_{cd} + \psi_a\sigma_b\overline{\psi}_{cd}\right)\right), \tag{35}$$

where $e \equiv \det(e^a_m)$, $\psi_{mn} \equiv \widetilde{D}_m\psi_n - \widetilde{D}_n\psi_m$, and $\widetilde{D}_m\psi_n \equiv (\partial_m + \omega_m)\psi_n$. The chiral superfield $\mathcal{E}$ is the SUSY extension of $e = \sqrt{-g}$, and $\mathcal{R}$ is the SUSY extension of the (Ricci) scalar curvature R . The real vector b_m and complex scalar X are known in the supergravity literature as the (old-minimal) set of fields needed to complete the off-shell supergravity multiplet with a closed algebra of SUSY transformations, i.e., independently upon an action.

其中 $e \equiv \det(e^a_m)$, $\psi_{mn} \equiv \widetilde{D}_m\psi_n - \widetilde{D}_n\psi_m$， 以及 $\widetilde{D}_m\psi_n \equiv (\partial_m + \omega_m)\psi_n$。手征超场 $\mathcal{E}$ 是 $e = \sqrt{-g}$ 的超对称 (SUSY) 扩展，而 $\mathcal{R}$ 是 (里奇) 标量曲率 R 的超对称扩展。在超引力文献中，实矢量场 b_m 和复标量场 X 是完成脱壳超引力多重态、满足超对称变换封闭代数所需的 (旧最小) 场集，也就是不依赖于作用量。

Single-field inflation is possible by identifying inflaton with one of the scalars while suppressing other scalars during inflation by assigning heavy masses to them (beyond the Hubble value). There is no need to learn supergravity theory for that because Eq. (33) is enough. There are several problems with this approach in supergravity. First, it is the so-called η -problem related to the e^K factor in the scalar potential that is too steep in the case of the canonical Kähler potential and does not have the required flatness during inflation or, equivalently, leads to a large slow-roll parameter η . Second, there is no good reason for a choice of the inflationary trajectory in the (scalar) multi-field space. Third, potential instabilities of the inflationary trajectory (because of inflaton mixing with other scalars) may easily spoil single-field inflation and thus fail to achieve the desired number of e-foldings. Though all those problems are solvable, it requires careful engineering of the potentials K and W , which implies rather low predictive power. Though a superpotential W is (perturbatively) protected against quantum corrections, it is not the case for a Kähler potential K .

单场暴涨可以通过如下方式实现: 将暴涨子认定为其中一个标量场，同时给其他标量场赋予远大于哈勃值的大质量，从而在暴涨过程中抑制这些标量场。对该问题而言，式 (33) 已经足够，无需学习完整的超引力理论。但超引力中的该方法存在若干问题: 第一，即所谓的 η 问题，它对标量势中的 e^K 因子相关，对于正则凯勒势而言该因子斜率过大，无法满足暴涨过程所需的平坦性，等价地说就是会得到过大的慢滚参数 η 。第二，在多标量场空间中选择暴涨轨迹没有合理的依据。第三，暴涨轨迹存在潜在不稳定性 (源于暴涨子与其他标量场的混合)，很容易破坏单场暴涨，无法得到所需的 e 折叠数。尽管这些问题都可以解决，但需要对势 K 和 W 进行精细设计，这意味着模型的预言能力很低。尽管超势 W (微扰论层面) 不受量子修正影响，但凯勒势 K 不具备这个性质。

It is therefore natural to adopt an economical approach in supergravity by minimizing a number of the physical degrees of freedom involved and by using the Starobinsky model as the starting point. Starobinsky-like supergravities can be introduced either as the locally supersymmetric extensions of the $\left(R+R^2\right)$ gravity action (5) or as the supergravity extensions of the quintessence model (4) with the inflaton potential (11). The modified supergravity models [42-44] can be reformulated in terms of the standard (Einstein) supergravity coupled to chiral and vector superfields, similar to the relation between modified $F(R)$ gravity and scalar-tensor gravity, which can also relate inflation to PBH and DM genesis. Several realizations of these ideas are described in the next sections.

因此，在超引力中采用简约方法是很自然的: 尽量减少涉及的物理自由度，并以斯塔罗宾斯基模型为出发点。类斯塔罗宾斯基超引力既可以作为 $\left(R+R^2\right)$ 引力作用量 (5) 的局域超对称推广引入，也可以作为带有暴胀子势 (11) 的精质模型 (4) 的超引力推广引入。修正超引力模型 [42-44] 可以用耦合手征超场与矢量超场的标准 (爱因斯坦) 超引力重新表述，这类似修正 $F(R)$ 引力与标量-张量引力之间的关系，该关系也能将暴胀与原初黑洞和暗物质起源联系起来。下文几节将介绍这些想法的几种实现方式。

Minimal Embedding of Inflation to Supergravity

暴胀向超引力的最小嵌入

In this section, the simplest minimal models of supergravity-based inflation are introduced, where (i) inflaton is assigned to a massive vector supermultiplet having only one physical scalar or (ii) the Starobinsky model (5) is embedded to supergravity by using a nilpotent chiral superfield having Volkov-Akulov goldstino.

本节将介绍基于超引力的最简单的暴胀最小模型，其中 (i) 暴胀子被赋予仅含一个物理标量的有质量矢量超多重态，或 (ii) 斯塔罗宾斯基模型 (5) 通过含有沃尔科夫-阿库洛夫戈德斯蒂诺的幂零手征超场嵌入超引力。

Single-Field Inflation in Supergravity

超引力中的单场暴涨

The minimal supergravity framework [45-48] allows one to embed any inflaton potential given by square of a real function. The inflaton field complexification in a chiral supermultiplet can be avoided by assigning inflaton to a massive vector supermultiplet V that has a single physical scalar. The scalar potential of a vector multiplet is given by the D-term instead of the F-term, while any desired values of the CMB observables (n_s and r) are possible. The manifestly supersymmetric Lagrangian is given by

最小超引力框架 [45-48] 允许我们嵌入任意由实函数平方给出的暴涨子势。可以通过将暴涨子分配给具有单个物理标量的有质量矢量超多重态 V，避免手征超多重态中暴涨子的复化。矢量多重态的标量势由 D 项而非 F 项给出，同时可以实现 CMB 可观测物理量 (n_s 和 r) 的任意期望取值。显式超对称拉格朗日量由下式给出

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left\{\frac{3}{8}\left(\overline{\mathcal{D}\mathcal{D}} - 8\mathcal{R}\right)e^{-\frac{2}{3}J} + \frac{1}{4}W^{\alpha}W_{\alpha}\right\} + \text{ h.c. }, \tag{36}$$

where $W_\alpha \equiv -\frac{1}{4}\left(\overline{\mathcal{D}}^2 - 8\mathcal{R}\right)\mathcal{D}_\alpha V$ is the Abelian superfield strength of the vector superfield V and J (V) is arbitrary real function.

其中 $W_\alpha \equiv -\frac{1}{4}\left(\overline{\mathcal{D}}^2 - 8\mathcal{R}\right)\mathcal{D}_\alpha V$ 是矢量超场 V 的阿贝尔超场强度，J (V) 是任意实函数。

The bosonic part (ignoring all contributions of fermions) of the Lagrangian in Einstein frame (after Weyl rescaling and elimination of the auxiliary fields) reads [45, 46]

爱因斯坦框架下 (外尔缩放并消去辅助场后) 拉格朗日量的玻色子部分 (忽略所有费米子贡献) 为 [45, 46]

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}J''\partial_m C\partial^m C - \frac{1}{2}J''B_m B^m - \frac{g^2}{2}J'^2, \tag{37}$$

where $C = V \mid$ is the real (scalar) inflaton field, $F_{mn} = \partial_m B_n - \partial_n B_m, J = J(C)$, and g is the coupling constant. The input for model building is given by a real function J (instead of K and W). Ghost freedom requires $J''(C) > 0$.

其中 $C = V \mid$ 是实 (标量) 暴涨子场 $F_{mn} = \partial_m B_n - \partial_n B_m, J = J(C)$，$g$ 是耦合常数。模型构建的输入由实函数 J 给出 (而非 K 和 W)。无鬼要求满足 $J''(C) > 0$。

For instance, the scalar potential of the Starobinsky inflation model is obtained by

例如，斯塔罗宾斯基暴涨模型的标量势可通过下式得到

$$J(C) = \frac{3}{2}(C - \ln C) \quad \text{and} \quad C = \exp\left(\sqrt{2/3}\varphi\right), \tag{38}$$

in terms of the canonical inflaton φ . In the case of the generalized alpha-attractors (23), one gets the nonlinear differential equations

用正则暴涨子 φ 表示。对于广义 α 吸引子 (23)，我们得到非线性微分方程

$$\frac{g}{\sqrt{2}}\frac{dJ}{dC} = f\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right) \quad \text{and} \quad \left(\frac{d\varphi}{dC}\right)^2 = \frac{d^2J}{dC^2}. \tag{39}$$

The master function $J(V)$ can be replaced by the function $\tilde{J}\left(He^{2V}\bar{H}\right)$, where we have introduced the (charged) Higgs chiral superfield H and have chosen $g = 1$ for simplicity. The arguments of the function $\tilde{J}$ and, hence, the function $\tilde{J}$ itself are both invariant under the gauge transformations

母函数 $J(V)$ 可以替换为函数 $\tilde{J}\left(He^{2V}\bar{H}\right)$，其中我们引入了 (带荷) 希格斯手征超场 H，为简化起见选择了 $g=1$。函数 $\tilde{J}$ 的自变量以及函数 $\tilde{J}$ 本身在规范变换下都不变

$$H \to e^{-iZ}H,\ \bar{H} \to e^{i\bar{Z}}\bar{H},\ V \to V + \frac{i}{2}\left(Z - \bar{Z}\right), \tag{40}$$

whose gauge parameter Z is also a chiral superfield. The original theory is recovered in the supersymmetric gauge $H = 1$.

其规范参数 Z 本身也是一个手征超场。原始理论可以在超对称规范 $H=1$ 中复原。

One can choose another (Wess-Zumino-type) supersymmetric gauge $V = V_1$, where V_1 describes the irreducible massless vector gauge supermultiplet minimally coupled to the dynamical Higgs-type chiral multiplet H . The standard super-Higgs mechanism [39] appears with the canonical function $J = \frac{1}{2}He^{2V}\bar{H}$ that corresponds to a linear function $\tilde{J}$ [47, 48] . In the case under consideration, the supersymmetric $U(1)$ gauge theory in terms of the superfields H and V_1 coupled to supergravity defines the analogue of Higgs inflation that is equivalent to the Starobinsky inflation by construction because both arise in the two different gauges of the same gauge theory. The chiral (Higgs) superfield H is charged with respect to $U(1)$, being the gauge degree of freedom that can be eaten up by the vector gauge supermultiplet becoming massive.

我们可以选择另一种 (维斯-朱米诺型) 超对称规范 $V=V_1$，其中 V_1 描述不可约无质量矢量规范超多重态，它与动力学希格斯型手征多重态 H 最小耦合。标准超希格斯机制 [39] 出现在对应线性函数 $\tilde{J}$ [47, 48] 的正则函数 $J = \frac{1}{2}He^{2V}\bar{H}$ 情形。在我们考虑的情况中，用耦合到超引力的超场 H 和 V_1 表示的超对称 $U(1)$ 规范理论定义了希格斯暴涨的对应形式，该形式按构造等价于斯塔罗宾斯基暴涨，因为二者源于同一规范理论的两种不同规范。手征 (希格斯) 超场 H 带 $U(1)$ 荷，作为规范自由度可以被矢量规范超多重态“吃掉”，使后者获得质量。

Given the inflaton potential (37), its minima are given by solutions to $J'(C) = 0$ because $J''(C) > 0$, that, in turn, implies only Minkowski vacua with restored SUSY and unrealistic phenomenology after inflation. Spontaneous SUSY breaking can be achieved by adding the so-called alternative Fayet-Iliopoulos terms [49, 50]; see section ”Polonyi-Starobinsky (PS) Supergravity”.

给定暴涨子势 (37)，其极小值是 $J'(C)=0$ 的解，因为 $J''(C)>0$，这反过来意味着暴涨后仅存在超对称恢复的闵氏真空，唯象学不符合实际。可以通过添加所谓的备选法耶特-伊柳普洛斯项实现自发超对称破缺 [49, 50]；参见“波洛尼-斯塔罗宾斯基 (PS) 超引力”一节。

Volkov-Akulov-Starobinsky Supergravity

沃尔科夫-阿库洛夫-斯塔罗宾斯基超引力

A generic embedding of inflation to supergravity requires two superfields, the one including inflaton φ and another one including goldstino G because of SUSY breaking. The number of physical degrees of freedom can be reduced by demanding the chiral goldstino superfield S to be nilpotent, $S^2 = 0$ [51, 52] . The expansion of S in terms of its field components reads

由于超对称破缺，将暴胀嵌入超引力的一般形式需要两个超场: 一个包含暴胀子 φ ，另一个包含金戈斯蒂诺 G 。我们可以要求手性金戈斯蒂诺超场 S 是幂零的 $S^2 = 0$ [51, 52] ，从而减少物理自由度的数量。S 按场分量的展开式为

$$S = \frac{GG}{2F} + \sqrt{2}\Theta G + \Theta^2 F, \tag{41}$$

where the leading scalar component is the goldstino bilinear GG .

其中领头标量分量是金戈斯蒂诺双线性 GG

The constrained goldstino superfield S is used to define the Volkov-Akulov-Starobinsky (VAS) supergravity by the setup [53]

我们利用带约束的金戈斯蒂诺超场 S ，通过如下构造定义沃尔科夫-阿库洛夫-斯塔罗宾斯基 (VAS) 超引力 [53]

$$K = -3\log\left(T + \bar{T} - \bar{S}S\right) \quad \text{and} \quad W = \lambda + \beta S + \gamma ST \tag{42}$$

with real parameters λ, β , and γ , the inflaton chiral superfield T , and the nilpotent goldstino superfield S . This construction is less minimal than that in the preceding subsection because the chiral superfield T includes besides inflaton φ its physical pseudo-scalar superpartner τ called sinflaton. The effective bosonic Lagrangian in the parametrization

其中包含实参数 λ, β 和 γ 、暴胀子手性超场 T 以及幂零金戈斯蒂诺超场 S 。该构造的最小性弱于前一小节的构造，因为手性超场 T 除包含暴胀子 φ 外，还包含其物理赝标超对称伙伴——西暴胀子 τ 。该参数化下的有效玻色子拉格朗日量为

$$T = \frac{T_0}{2}\left[e^{\sqrt{\frac{2}{3}}\varphi} + i\sqrt{\frac{2}{3}}\tau\right], \; T_0 = -2\beta/\gamma \equiv \langle T \rangle, \tag{43}$$

reads

形式如下

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\varphi}\left(\partial_\mu\tau\right)^2 - \frac{\gamma^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - \frac{\gamma^2}{18}e^{-2\sqrt{\frac{2}{3}}\varphi}\tau^2$$

(44)

and has a Minskowski minimum at $\varphi = 0$ with the Starobinsky potential and the formal masses $m_\varphi = m_\tau = \gamma/3$. During inflation when $\varphi \gg 1$, the sinflaton dynamics is suppressed by the exponential terms in Eq. (44). After inflation, when inflaton is settled at the minimum, SUSY is broken due to the nonvanishing vacuum expectation value of the F -field component of the inflaton superfield T , with the gravitino mass

它在 $\varphi=0$ 处存在闵氏极小值，对应斯塔罗宾斯基势，形式质量为 $m_\varphi=m_\tau=\gamma/3$。当 $\varphi\gg 1$、暴胀发生时，西暴胀子的动力学被式 (44) 中的指数项压制。暴胀结束后，暴胀子停在极小值处，由于暴胀子超场 T 的 F 场分量具有非零真空期望值，超对称发生破缺，引力微子质量为

$$m_{3/2}=\langle e^{K/2}|W|\rangle=\frac{\gamma^{3/2}\lambda}{2\sqrt{2}\beta^{3/2}}. \tag{45}$$

The nilpotency constraint is no longer valid at the minimum because the vacuum expectation value of the F -component of S becomes zero. It can be avoided by adding a linear term in T to the superpotential (42), leading to a de Sitter vacuum after inflation [54].

幂零约束在极小值处不再成立，因为 S 的 F 分量的真空期望值为零。我们可以通过在超势 (42) 中添加一个 T 的线性项来避免该问题，得到暴胀后的德西特真空 [54]。

The higher-derivative Starobinsky-like supergravity model dual to the ”quintessence” supergravity model (42) also exists [53,54]. The supersymmetric Lagrangian with the Kähler potential and the superpotential given in Eq. (42) read

还存在与“精质”超引力模型 (42) 对偶的高阶导数类斯塔罗宾斯基超引力模型 [53,54]。由式 (42) 给出凯勒势和超势的超对称拉格朗日量为

$$\begin{aligned}\mathcal{L}&=\int d^2\Theta 2\mathcal{E}\left[\frac{3}{8}\left(\overline{\mathcal{D}}^2-8\mathcal{R}\right)\left(T+\bar{T}-\bar{S}S\right)+\lambda+\beta S+\gamma ST\right]+\text{ h.c.} \\ &=\int d^2\Theta 2\mathcal{E}\left[-\frac{3}{8}\left(\overline{\mathcal{D}}^2-8\mathcal{R}\right)\bar{S}S+\lambda+\beta S-T\left(6\mathcal{R}-\gamma S\right)\gamma ST\right]+\text{ h.c.},\end{aligned} \tag{46}$$

where we have used the superspace identity [39]

此处我们用到了超空间恒等式 [39]

$$\int d^2\Theta 2\mathcal{E}\left(\overline{\mathcal{D}}^2-8\mathcal{R}\right)\left(T+\bar{T}\right)+\text{ h.c.}\ =-16\int d^2\Theta 2\mathcal{E}\mathcal{R}T+\text{ h.c.} \tag{47}$$

Varying the action (46) with respect to T yields

将作用量 (46) 对 T 变分可得

$$S=\frac{6}{\gamma}\mathcal{R} \tag{48}$$

and, hence, the scalar curvature chiral superfield $\mathcal{R}$ is also nilpotent, $\mathcal{R}^2=0$, because of $S^2=0$. Substituting the solution (48) back into the action (46) leads to the Starobinsky-like supergravity with the Lagrangian

因此标量曲率手性超场 $\mathcal{R}$ 也是幂零的 $\mathcal{R}^2=0$，这是因为 $S^2=0$。将解 (48) 代回作用量 (46)，即可得到拉格朗日量形式的类斯塔罗宾斯基超引力

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[-3\mathcal{R} - \frac{27}{2\gamma^2} \left(\overline{\mathcal{D}}^2 - 8\mathcal{R} \right) \overline{\mathcal{R}}\mathcal{R} + \lambda + \Lambda\mathcal{R}^2 \right] + \text{ h.c.} , \tag{49}$$

where we have fixed $\beta = -\gamma/2$ in order to get the proper normalization of the first (Einstein) supergravity term and have introduced the Lagrange multiplier chiral superfield Λ in order to enforce the nilpotency condition $\mathcal{R}^2 = 0$. This condition eliminates the scalar and pseudo-scalar degrees of freedom in the scalar curvature chiral superfield $\mathcal{R}$, the goldstino arises from the gauge-invariant gravitino field strength, and the axion is traded for the longitudinal mode $\mathcal{D} \cdot b$ of the pseudo-vector field b_m [53]. The parameter γ is fixed by the scalaron mass, while the parameter λ is related to the scale of SUSY breaking.

此处我们固定了 $\beta = -\gamma/2$ ，以得到第一个 (爱因斯坦) 超引力项的正确归一化，并且引入了拉格朗日乘子手性超场 Λ ，以施加幂零条件 $\mathcal{R}^2 = 0$ 。该条件消去了标量曲率手性超场 $\mathcal{R}$ 中的标量和赝标自由度，金戈斯蒂诺来自规范不变的引力微子场强，轴子则替换为赝矢量场 b_m 的纵向模式 $\mathcal{D} \cdot b$ [53]。参数 γ 由标量子质量固定，参数 λ 则与超对称破缺的能标相关。

Starobinsky-Type Supergravity and PBH Production

Starobinsky 型超引力与原初黑洞产生

The Starobinsky-type modified supergravity without nilpotent superfields is defined by the Lagrangian [43, 55]

不含幂零超场的 Starobinsky 型修正超引力由拉格朗日量 [43, 55] 定义

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[-\frac{1}{8} \left(\overline{\mathcal{D}}^2 - 8\mathcal{R} \right) N \left(\mathcal{R}, \overline{\mathcal{R}} \right) + \mathcal{F}\left(\mathcal{R} \right) \right] + \text{ h.c.} \tag{50}$$

with two arbitrary potentials $N\left(\mathcal{R}, \overline{\mathcal{R}}\right)$ (real) and $\mathcal{F}(\mathcal{R})$ (holomorphic), where $\mathcal{R}$ is the chiral scalar curvature superfield. The Lagrangian (50) is a generic off-shell (locally) supersymmetric extension of the modified $\left(R + R^2\right)$ gravity with four real scalars (including scalaron), all belonging to a single (off-shell) supergravity multiplet described by the chiral superfield $\mathcal{R}$ having the space-time scalar curvature R , the gravitino field strength, a real pseudo-vector b_m , and a complex scalar X as its field components. The fields b_m and X are known in the supergravity literature as the old-minimal set of the auxiliary fields needed to complete the supergravity multiplet with a closed algebra of SUSY transformations. In the modified (Starobinsky-type) supergravity, these "auxiliary" fields become dynamical or propagating because of the higher derivatives in the Lagrangian (50). No higher powers of R actually appear beyond the linear and quadratic terms. The first (D-type) term and the second (F-type) term in Eq. (50) are similar to the two terms in Eq. (29); however, they do not represent matter, being depended upon the supergravity fields only. The standard (Einstein) supergravity action [39] is the extension of the Einstein-Hilbert term linear in R , which is recovered in the special case of $N = 0$ and $\mathcal{F} = -3\mathcal{R}$.

它包含两个任意势: 实势 $N\left(\mathcal{R},\overline{\mathcal{R}}\right)$ 和全纯势 $\mathcal{F}(\mathcal{R})$，其中 $\mathcal{R}$ 是手征标量曲率超场。拉格朗日量 (50) 是修正 $\left(R+R^2\right)$ 引力的通用脱壳 (局部) 超对称推广，引入了四个实标量 (包括 scalaron)，全部属于单个脱壳超引力多重态，由手征超场 $\mathcal{R}$ 描述，其场分量包括时空标量曲率 R、引力微子场强、实伪矢量 b_m 和复标量 X。b_m 和 X 在超引力文献中被称为旧最小型辅助场集合，用于补全超引力多重态，得到封闭的超对称变换代数。在修正的 Starobinsky 型超引力中，这些“辅助”场因拉格朗日量 (50) 中的高阶导数变为动力学场，即可以传播。实际上，R 除一次项和二次项外不会出现更高次幂。式 (50) 中第一项 D 型项和第二项 F 型项与式 (29) 中的两项形式类似，但它们不描述物质，仅依赖超引力场。标准爱因斯坦超引力作用量 [39] 是 R 的线性爱因斯坦-希尔伯特项的推广，在 $N=0$ 和 $\mathcal{F}=-3\mathcal{R}$ 的特殊情况下可以回到该形式。

Let us expand the functions N and $\mathcal{F}$ in Taylor series and keep only a few leading terms as [56]

我们将函数 N 和 $\mathcal{F}$ 展开为泰勒级数，仅保留前几阶项，得到 [56]

$$N=\frac{12}{M^2}|\mathcal{R}|^2-\frac{72}{M^4}\zeta|\mathcal{R}|^4-\frac{768}{M^6}\gamma|\mathcal{R}|^6, \tag{51}$$

$$\mathcal{F}=-3\mathcal{R}+\frac{3\sqrt{6}}{M}\delta\mathcal{R}^2, \tag{52}$$

where M is the scalaron mass, with the parameters ζ,γ, and δ. The M^2 enters as the overall factor in the scalar potential and thus does not change its shape. In the case of $\zeta=\gamma=\delta=0$, one gets the simplest supersymmetric extension of $\left(R+R^2\right)$ gravity. However, it leads to a tachyonic instability along the inflationary trajectory and the scalar potential unbounded from below. This can be avoided by introducing the extra term $\zeta|\mathcal{R}|^4$ as in Eq. (51), whose parameter has the lower bound [44,57]. The model (50) with $\gamma=\delta=0$ and $\zeta>1/54$ is known the simplest phenomenologically viable extension of Starobinsky inflation in the old-minimal supergravity without nilpotent superfields [58].

其中 M 是 scalaron 质量，参数为 ζ,γ 和 δ。M^2 作为标量势的整体因子出现，因此不会改变势的形状。当 $\zeta=\gamma=\delta=0$ 时，我们得到 $\left(R+R^2\right)$ 引力最简单的超对称推广，但该情况会导致暴胀轨迹上存在快子不稳定性，且标量势无下界。这种问题可以通过引入式 (51) 中的额外项 $\zeta|\mathcal{R}|^4$ 避免，该参数有下界 [44,57]。取 $\gamma=\delta=0$ 和 $\zeta>1/54$ 的模型 (50) 是不含幂零超场的旧最小超引力中，Starobinsky 暴胀最简单且唯象上可行的推广 [58]。

By extending the model further, either via N with $\gamma\neq 0,\delta=0$ or via $\mathcal{F}$ with $\gamma=0,\delta\neq 0$, it is possible to achieve an enhancement of the power spectrum of scalar perturbations at a scale smaller the inflationary scale, which is necessary to produce PBH seeds [56]. Focusing on the effective dynamics of two real scalars (when the others are stabilized), the enhancement of the power spectrum is achieved due to a saddle point in the two-field scalar potential, which creates a short period of the ”ultra-slow-roll” (USR) inflation (actually, during USR, inflaton rolls faster than during SR [59]). The USR regime leads to a violation of the slow-roll conditions. The SR stage is driven by scalaron, whereas the USR stage is driven by a combination of both scalars.

通过进一步扩展模型，无论是通过 N 与 $\gamma \neq 0, \delta = 0$ ，还是通过 $\mathcal{F}$ 与 $\gamma = 0, \delta \neq 0$ ，都可以在小于暴胀能标的尺度上实现标量扰动功率谱增强，这是产生原初黑洞种子的必要条件 [56]。当聚焦于两个实标量的有效动力学 (其他标量已被稳定) 时，功率谱增强源于双场标量势中的鞍点: 鞍点会产生一段短时期的“超慢滚”(USR) 暴胀 (实际上，超慢滚阶段的暴胀子滚动速度比慢滚阶段更快 [59])。超慢滚阶段会破坏慢滚条件。慢滚阶段由 scalaron 驱动，而超慢滚阶段由两个标量共同驱动。

Let us call the model with $\gamma \neq 0$ and $\delta = 0$ as the γ -extension and the model with $\delta \neq 0$ and $\gamma = 0$ as the δ -extension. According to Ref. [56], the γ -extension exhibits the attractor behavior, in the sense that the shape of the scalar potential becomes less sensitive to changes in γ when the value of γ increases. The enhancement of the power spectrum can be achieved when $\gamma \geq \mathcal{O}(1)$ and ζ is tuned around the saddle point value in order to control the duration of USR stage ΔN_2 - the longer it lasts, the larger the power spectrum peak grows. As for the δ -extension, no attractor behavior is found, though the desired power spectrum peak is possible in the two parameter regions - the one is around $\delta = 0.1$, and another one is around $\delta = 0.6-$ while the parameter ζ controls the duration of the USR stage here as well.

我们将含 $\gamma \neq 0$ 和 $\delta = 0$ 的模型称为 γ 扩展，将含 $\delta \neq 0$ 和 $\gamma = 0$ 的模型称为 δ 扩展。根据文献 [56], γ 扩展具有吸引子行为: 当 γ 的取值增大时，标量势的形状对 γ 变化的敏感度会降低。通过在鞍点值附近调节 $\gamma \geq \mathcal{O}(1)$ 和 ζ 来控制超慢滚 (USR) 阶段 ΔN_2 的持续时间，即可实现功率谱增强——该阶段持续时间越长，功率谱峰就越大。而 δ 扩展不存在吸引子行为，但仍可在两个参数区域得到期望的功率谱峰: 一个区域在 $\delta = 0.1$ 附近，另一个区域在 $\delta = 0.6-$ 附近，这里同样由参数 ζ 控制 USR 阶段的持续时间。

The relevant part of the Lagrangian is calculable by parametrizing the leading field component of the curvature superfield as

拉格朗日量的相关部分可通过对曲率超场的领头场分量参数化计算得到:

$$\mathcal{R}|_{\Theta=0} = \frac{M}{\sqrt{24}} e^{-ia}\sigma, \tag{53}$$

and setting $b_m = a = 0$, where b_m is the real vector of an old-minimal supergravity multiplet and the real scalars σ and a are the radial and angular modes of $\mathcal{R}\,|$, respectively. After using the standard Legendre-Weyl transform to eliminate the R^2 - term, the bosonic part (ignoring all contributions of fermions) of the Lagrangian in the Einstein frame reads

并设置 $b_m = a = 0$ ，其中 b_m 是旧极小超重力多重态的实矢量，实标量 σ 和 a 分别是 $\mathcal{R}\,|$ 的径向模和角模。通过标准勒让德-外尔变换消去 R^2 项后，爱因斯坦框架下拉格朗日量的玻色子部分 (忽略所有费米子贡献) 可写为:

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - \frac{3M^2}{2}Be^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U,$$

(54)

where φ is the scalaron and the functions $A \equiv A(\sigma)$, $B \equiv B(\sigma)$, and $U \equiv U(\sigma)$ are given by

其中 φ 是标量子，函数 $A \equiv A(\sigma), B \equiv B(\sigma)$ 和 $U \equiv U(\sigma)$ 由下式给出:

$$A(\sigma) = 1 - \delta\sigma + \frac{1}{6}\sigma^2 - \frac{11}{24}\zeta\sigma^4 - \frac{29}{54}\gamma\sigma^6,$$

$$B(\sigma) = \frac{1}{3}M^{-2}\left(1 - \zeta\sigma^2 - \gamma\sigma^4\right), \tag{55}$$

$$U(\sigma) = \frac{1}{2}M^2\sigma^2\left(1 + \frac{1}{2}\delta\sigma - \frac{1}{6}\sigma^2 + \frac{3}{8}\zeta\sigma^4 + \frac{25}{54}\gamma\sigma^6\right).$$

The Kähler potential and the superpotential of the Einstein supergravity dual to the modified supergravity defined by Eqs. (51) and (52) are given by

对应于由式 (51) 和 (52) 定义的修正超引力，爱因斯坦超引力的凯勒势和超势为:

$$K = -3\log\left[T + \bar{T} - \frac{1}{3}N\left(S, \bar{S}\right)\right], \; W = 3MST + \mathcal{F}(S), \tag{56}$$

where T and S are the chiral (super)fields, and the functions

其中 T 和 S 是手征 (超) 场，且函数

$$N\left(S, \bar{S}\right) = 3\left(|S|^2 - \frac{3}{2}\zeta|S|^4 - 4\gamma|S|^6\right), \; \mathcal{F}(S) = 3MS\left(\frac{\sqrt{6}}{4}\delta S - \frac{1}{2}\right),$$

(57)

have been obtained from Eqs. (51) and (52) by replacing $\mathcal{R} = MS/2$. In particular, Eq. (53) gives $S = e^{-ia}\sigma/\sqrt{6}$. The scalaron φ in the dual picture is given by

可由式 (51) 和 (52) 替换 $\mathcal{R} = MS/2$ 得到。特别地，式 (53) 给出 $S = e^{-ia}\sigma/\sqrt{6}$ 。对偶图景中标量子 φ 由下式给出:

$$e^{\sqrt{\frac{2}{3}}\varphi} = T + \bar{T} - \frac{1}{3}N\left(S, \bar{S}\right). \tag{58}$$

Setting $\mathrm{Im}\, T = a = 0$ gives the Lagrangian (54).

取 $\mathrm{Im}\, T = a = 0$ 即可得到拉格朗日量 (54)。

The mass of PBH created as a result of the primordial power spectrum enhancement followed by gravitational collapse of large density perturbations can be estimated from the peak as follows [23]:

原初功率谱增强引发大密度扰动引力塌缩产生的原初黑洞质量，可通过峰按下式估计 [23]:

$$M_{\mathrm{PBH}} \simeq \frac{M_{\mathrm{Pl}}^2}{H\left(t_{\mathrm{peak}}\right)} \exp\left[2\left(N_{\mathrm{end}} - N_{\mathrm{peak}}\right) + \int_{t_{\mathrm{peak}}}^{t_{60}} \varepsilon(t) H(t)\, dt\right], \tag{59}$$

where t_{peak} is the time when the wavenumber corresponding to the power spectrum peak (k_{peak}) exits the horizon and t_{60} is the time when k_{60} exits the horizon.

其中 t_{peak} 是功率谱峰 (k_{peak}) 对应的波数出视界的时刻，t_{60} 是 k_{60} 出视界的时刻。

The PBH density fraction in DM can be roughly estimated by using the standard (Press-Schechter) formalism [60]. The underlying formulae for the PBH mass $\widetilde{M}_{\text{PBH}}(k)$, the production rate $\beta_f(k)$, and the density contrast $\sigma(k)$ coarse-grained over k are [61, 62]

原初黑洞在暗物质中的密度占比可通过标准 (Press-Schechter) 形式论粗略估计 [60]。对应原初黑洞质量 $\widetilde{M}_{\text{PBH}}(k)$、产生率 $\beta_f(k)$ 和在 k 上粗粒化的密度反差 $\sigma(k)$ 的基本公式为 [61, 62]

$$\widetilde{M}_{\text{PBH}} \simeq 10^{20}\left(\frac{7\times 10^{12}}{k\text{Mpc}}\right)^2 \text{g},\ \beta_f(k) \simeq \frac{\sigma(k)}{\sqrt{2\pi}\delta_c} e^{-\frac{\delta_c^2}{2\sigma^2(k)}}, \tag{60}$$

$$\sigma^2(k) = \frac{16}{81}\int \frac{dq}{q}\left(\frac{q}{k}\right)^4 e^{-q^2/k^2} P_\zeta(q),$$

respectively, where the parameter δ_c is the density threshold for PBH formation. The PBH mass is estimated as the horizon mass at the time when the co-moving momentum k reenters the horizon. The PBH-to-DM density fraction is given by [61,62]

其中参数 δ_c 是原初黑洞形成的密度阈值。原初黑洞质量等于共动动量 k 重新进入视界时的视界质量。原初黑洞与暗物质的密度占比由下式给出 [61,62]:

$$\frac{\Omega_{\text{PBH}}(k)}{\Omega_{\text{DM}}} \equiv f(k) \simeq \frac{1.4\times 10^{24}\beta_f(k)}{\sqrt{\widetilde{M}_{\text{PBH}}(k)\,\text{g}^{-1}}}. \tag{61}$$

Three specific examples in the Starobinsky supergravity were proposed and studied in Ref. [56]: one γ -extension and two δ -extensions. The δ -extensions were motivated by the existence of two suitable parameter regions, where $\delta \simeq 0.1$ and $\delta \simeq 0.6$ yield different shapes of the power spectrum (broad and narrow, respectively). The parameter sets of all three examples are given in Table 1, and the corresponding power spectra P_ζ and PBH density fractions $f(M)$ (numerically computed in Ref. [56]) are shown in Fig. 1, with the normalization of the wavenumber $k_{\text{exit}} = 0.05\text{Mpc}^{-1}$, where k_{exit} is the scale that leaves the horizon around 54e-folds before the end of inflation. The parameter ζ is fixed by the choice of ΔN_2 at given γ and δ. In cases I, II, and III, one finds ζ as -2.374, 0.032, and 0.102, respectively.

文献 [56] 在斯塔罗宾斯基超引力中提出并研究了三个具体例子:1 个 γ 扩展和 2 个 δ 扩展。δ 扩展的研究动机是存在两个合适的参数区域，其中 $\delta \simeq 0.1$ 和 $\delta \simeq 0.6$ 分别产生宽型和窄型两种不同形状的功率谱。三个例子的参数集均列于表 1，对应功率谱 P_ζ 和原初黑洞密度分数 $f(M)$ (由文献 [56] 数值计算得到) 展示于图 1，波数按 $k_{\text{exit}} = 0.05\text{Mpc}^{-1}$ 归一化，其中 k_{exit} 是 inflation 结束前约 54 个 e 折叠时离开视界的尺度。参数 ζ 由给定 γ 和 δ 下 ΔN_2 的选择固定。在案例 I、II、III 中，ζ 分别为-2.374、0.032 和 0.102。

To demonstrate the end of SR and the beginning of USR, Fig. 2 shows the evolution of the SR parameters ε_H and η_H in the case II. The SR parameters are defined by

为展示慢滚 (SR) 的结束和超慢滚 (USR) 的开始，图 2 给出了案例 II 中慢滚参数 ε_H 和 η_H 的演化。慢滚参数定义为

$$\varepsilon_H \equiv -\frac{\dot{H}}{H^2},\ \eta_H \equiv \frac{\dot{\varepsilon}_H}{H\varepsilon_H}. \tag{62}$$

Table 1 The selected parameter values and the corresponding CMB tilts n_s and r computed at $\Delta N = 54$ e-folds before the end of inflation (including the USR e-folds ΔN_2)

表 1 所选参数值，以及在 inflation 结束前 $\Delta N = 54$ 个 e 折叠处计算得到的对应 CMB 倾斜 n_s 和 r (包含超慢滚 e 折叠 ΔN_2)

	Y	δ	ΔN_2	δ_c	n_s	r
Case I	1.5	0	20	0.4	0.942	0.009
Case II	0	0.09	19	0.47	0.946	0.008
Case III	0	0.61	20	0.4	0.946	0.007

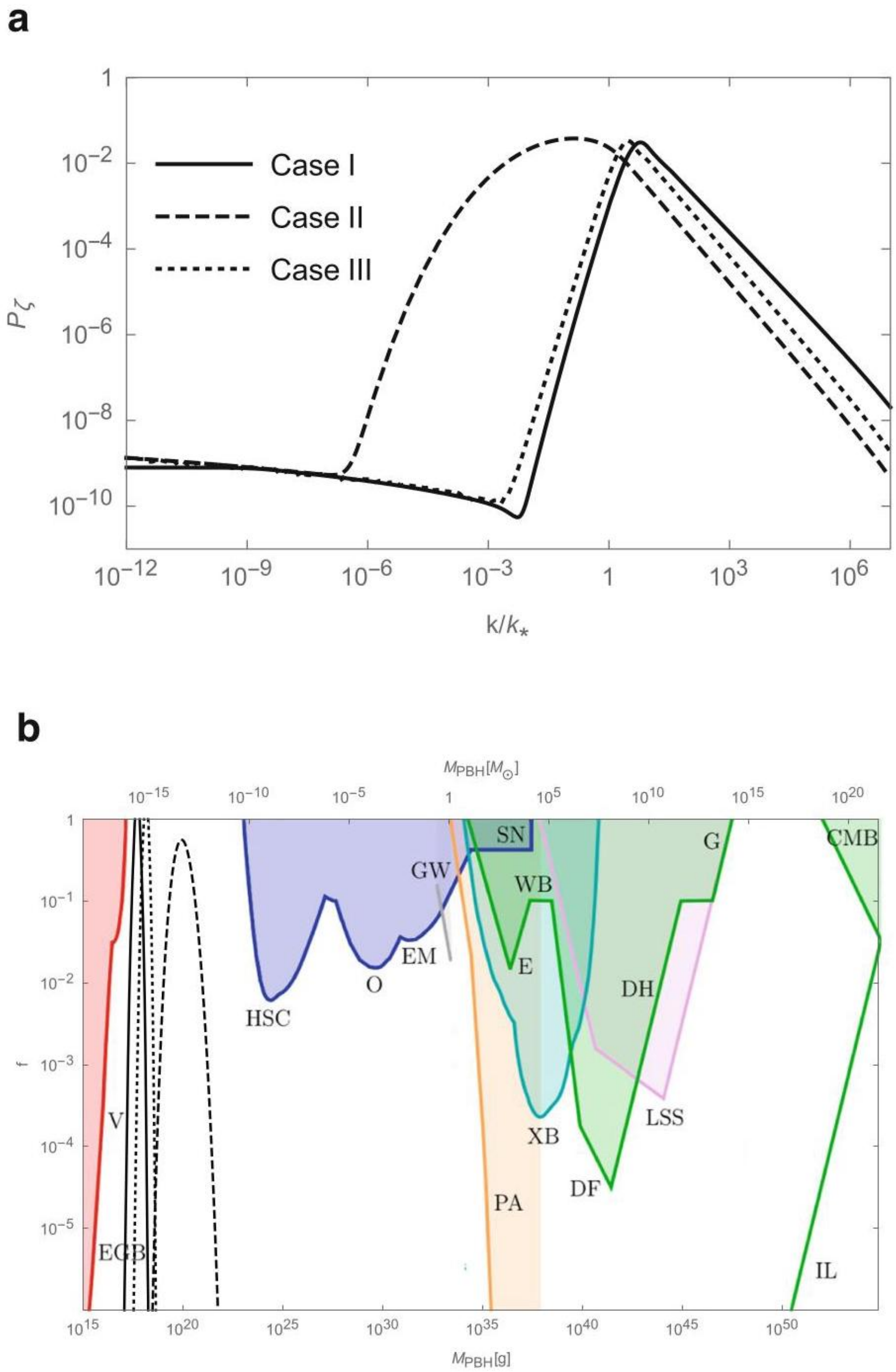

Fig. 1 (a) The power spectra in the examples of Table 1. Here, k_* represents the end of SR and the beginning of USR. (b) The respective PBH density fractions, where the background observational constraints on PBH are taken from Ref. [12]. In both plots, case I is denoted by the solid line, case II by the dashed line, and case III by the dotted line

图 1 (a) 表 1 中各例子的功率谱。此处 k_* 代表慢滚结束、超慢滚开始的位置。(b) 对应原初黑洞密度分数，原初黑洞的背景观测约束取自文献 [12]。两幅图中，案例 I 用实线表示，案例 II 用虚线表示，案例 III 用点线表示

The end of SR can be defined by the local maximum of ε_H (or, alternatively, by $\eta_H = 1$), and it is shown in Fig. 2 by the dashed vertical line.

慢滚的结束可定义为 ε_H 的局部最大值 (或等价地，由 $\eta_H = 1$ 定义)，在图 2 中用竖直虚线标出。

According to Table I, the spectral tilt n_s in case I is ruled out by 3σ from the CMB data [29], whereas in cases II and III the value of n_s is within the current 3σ constraints. The PBH fraction in case II of Fig. 1 implies that this case is more flexible for accommodating slightly larger n_s . It happens because the PBH fraction in case II peaks at the center of the allowed window, and it is still possible to move the peak further to the left, thus lowering the PBH masses. More general cases with the nonvanishing γ and δ do not increase $M_{\rm PBH}$ and n_s [63].

根据表 1，案例 I 中的谱倾斜 n_s 被 CMB 数据 [29] 给出的 3σ 排除，而案例 II 和 III 中 n_s 的值符合当前 3σ 的约束范围。图 1 案例 II 中的原初黑洞占比说明该案例更易容纳稍大的 n_s 。这是因为案例 II 中原初黑洞占比的峰值位于允许窗口的中心，仍可将峰值进一步左移，从而得到更小的原初黑洞质量。非零 γ 和 δ 的更一般情况不会增大 $M_{\rm PBH}$ 和 n_s [63]。

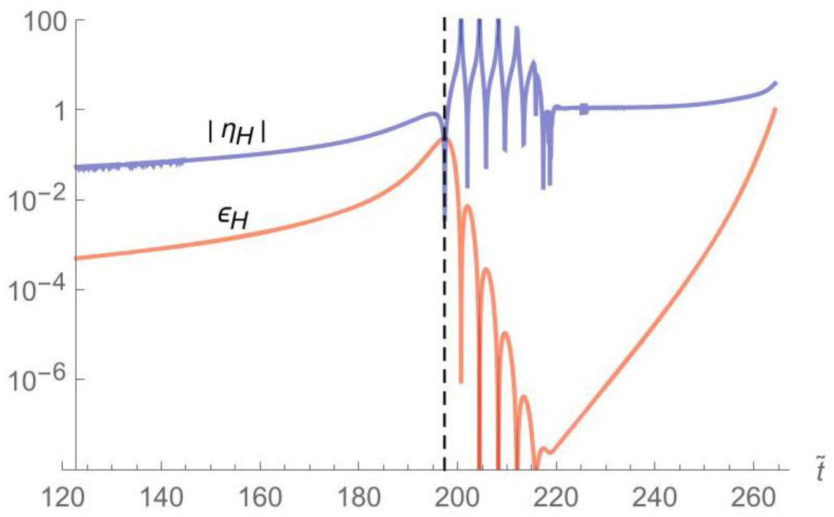

Fig. 2 The evolution of the slow-roll parameters ε_H and $|\eta_H|$ in case II around the start of the USR regime with respect to the normalized time $\tilde{t}$

图 2 超慢滚区域起始附近，案例 II 中慢滚参数 ε_H 和 $|\eta_H|$ 随归一化时间 $\tilde{t}$ 的演化

Induced Gravitational Waves

诱导引力波

In order to confirm or falsify the proposed supergravity models of inflation and PBH production by observations, one may look at detection of the stochastic gravitational waves (GW) induced by PBH formation. Let us estimate the energy density of the induced GW in the examples of Table 1 by following Ref. [64].

为了通过观测验证或证伪已提出的暴胀超引力模型与原初黑洞产生模型，我们可以研究探测由原初黑洞形成诱导产生的随机引力波 (GW)。下面我们沿用文献 [64] 的方法，估计表 1 例子中诱导引力波的能量密度。

The present-day GW density function (spectrum) $\Omega_{\rm GW}$ is given by [65,66]

当前引力波密度函数 (能谱) $\Omega_{\rm GW}$ 由文献 [65,66] 给出

$$\frac{\Omega_{\rm GW}(k)}{\Omega_r} = \frac{c_g}{72}\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} {\rm d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} {\rm d}s \left[\frac{\left(s^2-\frac{1}{3}\right)\left(d^2-\frac{1}{3}\right)}{s^2+d^2}\right]^2 P_\zeta(kx) P_\zeta(ky)\left(I_c^2+I_s^2\right),$$

(63)

where the constant $c_g \approx 0.4$ in the case of the standard model (SM) and $c_g \approx 0.3$ in the case of the minimal supersymmetric standard model (MSSM); see Ref. [67] for details.

其中标准模型 (SM) 情况下常数为 $c_g \approx 0.4$ ，最小超对称标准模型 (MSSM) 情况下常数为 $c_g \approx 0.3$ ；详见文献 [67]。

The present-day value of the radiation density Ω_r is equal to $h^2\Omega_r \approx 2.47\times 10^{-5}$, according to measurements of CMB temperature [68]. Here, h is the reduced (present-day) Hubble parameter that we take as $h = 0.67$ (ignoring the Hubble tension). The variables x, y are related to the integration variables s, d as

根据宇宙微波背景温度的测量结果 [68]，当前辐射密度 Ω_r 的值等于 $h^2\Omega_r \approx 2.47\times 10^{-5}$ 。式中 h 是约化 (当前) 哈勃参数，我们取其为 $h = 0.67$ (忽略哈勃张力)。变量 x, y 与积分变量 s, d 满足关系

$$x = \frac{\sqrt{3}}{2}(s+d), \quad y = \frac{\sqrt{3}}{2}(s-d), \tag{64}$$

while the functions I_c and I_s of $x(s,d)$ and $y(s,d)$ are given by [65,66]

关于 $x(s,d)$ 和 $y(s,d)$ 的函数 I_c 和 I_s 由文献 [65,66] 给出

$$I_c = -4\int_0^\infty {\rm d}\eta \sin\eta \{2T(x\eta)T(x\eta) + [T(x\eta) + x\eta T'(x\eta)][T(y\eta) + y\eta T'(y\eta)]\}$$

(65)

and

以及

$$I_s = 4\int_0^\infty {\rm d}\eta \cos\eta \{2T(x\eta)T(x\eta) + [T(x\eta) + x\eta T'(x\eta)][T(y\eta) + y\eta T'(y\eta)]\},$$

(66)

where the T -function is defined by

其中 T 函数定义为

$$T(k\eta) = \frac{9}{(k\eta)^2}\left[\frac{\sqrt{3}}{k\eta}\sin\left(\frac{k\eta}{\sqrt{3}}\right) - \cos\left(\frac{k\eta}{\sqrt{3}}\right)\right], \tag{67}$$

in terms of the conformal time η .

用共形时间 η 表示。

The integrations in I_c and I_s can be performed analytically, and the results are [65]

I_c 和 I_s 中的积分可以解析求解，结果见文献 [65]

$$I_c = -36\pi\frac{\left(s^2+d^2-2\right)^2}{\left(s^2-d^2\right)^3}\theta(s-1), \tag{68}$$

$$I_s = -36\frac{s^2+d^2-2}{\left(s^2-d^2\right)^2}\left[\frac{s^2+d^2-2}{s^2-d^2}\log\left|\frac{d^2-1}{s^2-1}\right|+2\right], \tag{69}$$

where θ is the Heaviside step function.

其中 θ 是海维赛德阶跃函数。

Using the formulae above, the GW density can be numerically computed from a given power spectrum. Let us consider the power spectra in the cases of Table 1, where PBH are merely a part of DM because the cases with $f_{\text{tot}} = 1$ have quite similar power spectra though with slightly larger peaks. By using the power spectra of Fig. 1 (on the left side), the density $\Omega_{\text{GW}}(k)$ in terms of frequency $k = 2\pi f$ is plotted in Fig. 3 together with the expected sensitivity curves for several space-based GW experiments planned in the future. Here, the power-law integrated curves [69] have been used and applied to the LISA noise model [70,71]. To draw the sensitivity curves, the parameters and the noise models for TianQin [72], Taiji [73], and DECIGO [74] have been used.

利用上述公式，可以给定功率谱数值计算引力波密度。我们考察表 1 中各情况的功率谱，这些情况中原初黑洞仅构成暗物质的一部分，因为 $f_{\text{tot}} = 1$ 对应的情况功率谱非常相似，仅峰值稍大。利用图 1(左侧) 的功率谱，密度 $\Omega_{\text{GW}}(k)$ 随频率 $k = 2\pi f$ 的变化绘制在图 3 中，同时图中还给出了多个未来计划空间引力波实验的预期灵敏度曲线。这里我们采用幂律积分曲线 [69]，并应用到 LISA 噪声模型 [70,71] 中。灵敏度曲线的绘制使用了天琴 [72]、太极 [73] 和 DECIGO[74] 的参数与噪声模型。

It follows from Fig. 3 that the upcoming space-based GW experiments are expected to be sensitive enough to detect the stochastic GW background predicted by some inflation models proposed in the preceding sections, where PBH may account for a significant fraction (or all) of DM. Figure 3 shows that the supergravity models produce GW peaking in the frequency range $10^{-3}\nabla\cdot10^{-1}$ Hz expected to be accessible by LISA, TianQin, Taiji, and DECIGO gravitational interferometers.

从图 3 可以看出，即将建成的空间引力波实验灵敏度足够探测前文部分暴胀模型预言的随机引力波背景，在这些模型中原初黑洞可以占暗物质的很大比例 (甚至全部)。图 3 表明，超引力模型产生的引力波峰值落在 $10^{-3}\nabla \cdot 10^{-1}$ Hz 频率范围，该范围恰好可被 LISA、天琴、太极和 DECIGO 引力波干涉仪探测。

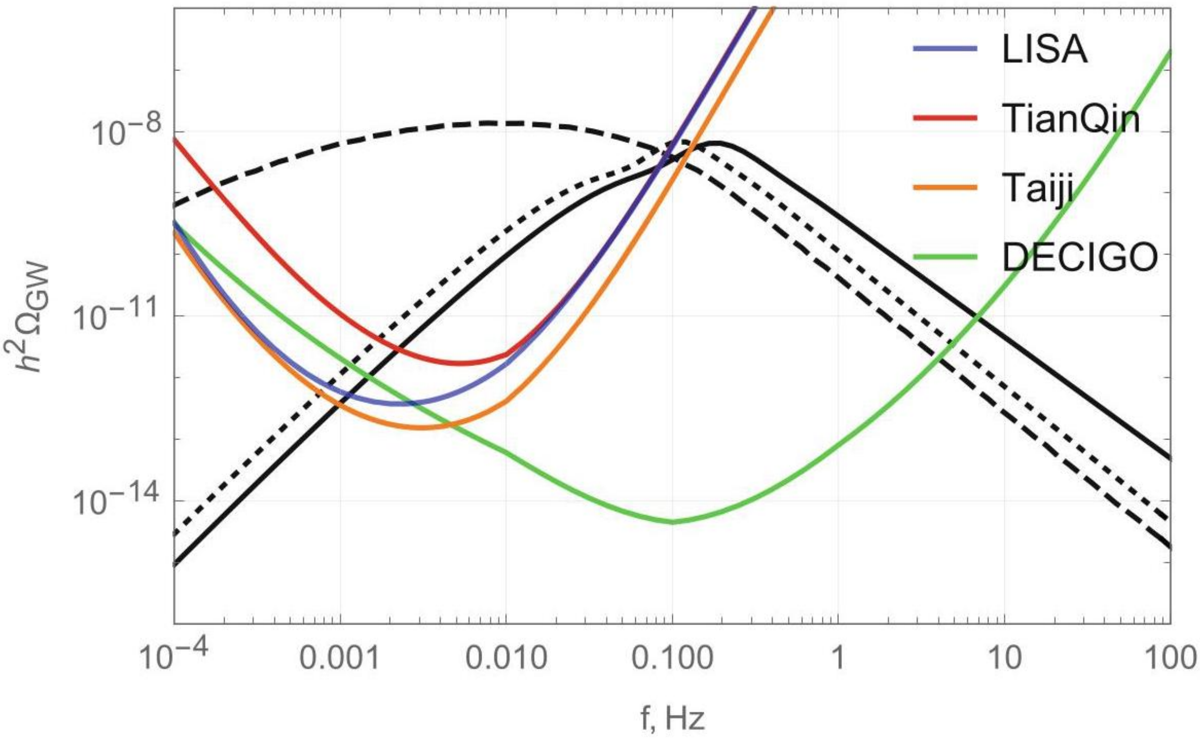

Fig. 3 The density of stochastic GW induced by the power spectrum enhancement in the supergravity models: case I (solid black curve), case II (dashed black curve), and case III (dotted black curve). The expected sensitivity curves for the space-based GW experiments are represented by the different colors

图 3 超引力模型中功率谱放大诱导的随机引力波密度: 情况 I(黑色实线)，情况 II(黑色虚线)，情况 III(黑色点线)。不同颜色代表各空间引力波实验的预期灵敏度曲线

Adding Matter and Spontaneous SUSY Breaking

添加物质与自发超对称破缺

The Starobinsky-type supergravity models considered in section ”Starobinsky-Type Supergravity and PBH Production” have only Minkowski vacua where SUSY is restored. An improved model of the Starobinsky-type supergravity coupled to a chiral matter superfield can simultaneously describe inflation, PBH formation, present DM, and spontaneous SUSY breaking after inflation in a Minkowski vacuum [75].

在“斯塔罗宾斯基型超引力与原初黑洞产生”一节中讨论的斯塔罗宾斯基型超引力模型仅拥有超对称恢复的闵氏真空。耦合手征物质超场的改进型斯塔罗宾斯基型超引力模型，可以同时描述暴胀、原初黑洞形成、当前暗物质，以及暴胀后闵氏真空中的自发超对称破缺 [75]。

The manifestly supersymmetric curved superspace Lagrangian of a chiral matter superfield Φ coupled to the modified Starobinsky-type supergravity is given by

耦合到改进型斯塔罗宾斯基型超引力的手征物质超场 Φ 的显超对称弯曲超空间拉格朗日量由下式给出

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left[-\frac{1}{8}\left(\overline{\mathcal{D}}^2 - 8\mathcal{R}\right)(N + J) + \mathcal{F} + \Omega\right] + \text{ h.c.} \tag{70}$$

It is parametrized by four arbitrary potentials: two non-holomorphic ones, $N = N\left(\mathcal{R}, \overline{\mathcal{R}}\right)$ and $J = J\left(\Phi, \overline{\Phi}\right)$, and two holomorphic ones, $\mathcal{F} = \mathcal{F}(\mathcal{R})$ and $\Omega = \Omega(\Phi)$, being the functions of the chiral scalar curvature superfield $\mathcal{R}$ of supergravity and the chiral superfield Φ of matter. When Φ enters J and Ω without mixing with $\mathcal{R}$, it is called the minimal coupling of the modified supergravity to chiral matter, as in Eq. (70).

它由四个任意势参数化: 两个非全纯势 $N = N\left(\mathcal{R}, \overline{\mathcal{R}}\right)$ 和 $J = J\left(\Phi, \overline{\Phi}\right)$，以及两个全纯势 $\mathcal{F} = \mathcal{F}(\mathcal{R})$ 和 $\Omega = \Omega(\Phi)$，它们都是超引力手征标量曲率超场 $\mathcal{R}$ 和物质手征超场 Φ 的函数。当 Φ 进入 J 和 Ω 且不与 $\mathcal{R}$ 混合时，这被称为改进超引力与手征物质的最小耦合，如式 (70) 所示。

Eliminating the auxiliary F-field of Φ in terms of the leading scalar field components $\phi \equiv \Phi \mid$ and $X \equiv \mathcal{R} \mid$ yields

用领头标量场分量 $\phi \equiv \Phi \mid$ 和 $X \equiv \mathcal{R} \mid$ 消去 Φ 的辅助 F 场后可得

$$F = -J^{-1}_{,\phi\bar{\phi}}\left(2\bar{X}J_{,\bar{\phi}} + \overline{\Omega}_{,\bar{\phi}}\right), \tag{71}$$

where the subscripts with commas denote the derivatives with respect to the given (field) arguments.

其中带逗号的下标表示对给定 (场) 变量求导。

The simplest model of SUSY breaking in the Einstein supergravity with a single chiral matter superfield is known as the Polonyi model where supergravity is minimally coupled to a chiral matter superfield Φ with the canonical kinetic term and the superpotential given by a linear polynomial in Φ. It leads to a Minkowski vacuum with spontaneously broken SUSY at any scale. In phenomenological applications, the Polonyi superfield is usually affiliated with the hidden (heavy) matter sector that interacts with the observable matter (like the SM) only gravitationally. Then SUSY breaking is supposed to be transferred to the observable sector at the electroweak scale by gravity mediation.

爱因斯坦超引力中单个手征物质超场实现超对称破缺的最简单模型被称为波洛尼模型，其中超引力与手征物质超场 Φ 最小耦合，该模型具有正则动能项，且超势是关于 Φ 的线性多项式。它在任意尺度下都能得到具有自发破缺超对称的闵氏真空。在唯象应用中，波洛尼超场通常归属于隐藏 (重) 物质部分，它仅通过引力与可观测物质 (如标准模型粒子) 相互作用。因此超对称破缺被认为通过引力中介传递到电弱尺度的可观测部分。

The bosonic part (ignoring all contributions of fermions) of the Lagrangian reads [75]

拉格朗日量的玻色子部分 (忽略所有费米子贡献) 可写为 [75]

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\text{bos.}} = & -\frac{1}{12}\left(\mathcal{F}_{,X}+\overline{\mathcal{F}}_{,\bar{X}}+2N_{,X}X+2N_{,\bar{X}}\bar{X}-8N_{,X\bar{X}}X\bar{X}+2N+2J-\frac{1}{9}N_{,X\bar{X}}b_m b^m\right)R \\
& +\frac{1}{144}N_{,X\bar{X}}R^2-N_{,X\bar{X}}\partial X\partial\bar{X}-J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi}-\frac{i}{3}b_m\left(N_{,X}\partial^m X+J_{,\phi}\partial^m\phi-\text{ c.c. }\right) \\
& +\frac{i}{6}\left(\mathcal{F}_{,X}-\overline{\mathcal{F}}_{,\bar{X}}+2N_{,X}X-2N_{,\bar{X}}\bar{X}-\frac{i}{6}N_{,X\bar{X}}D_m b^m\right)D_m b^m \\
& -\frac{1}{2}\left(\mathcal{F}_{,X}+\overline{\mathcal{F}}_{,\bar{X}}+2N_{,X}X+2N_{,\bar{X}}\bar{X}-4N_{,X\bar{X}}X\bar{X}+2N+2J\right. \\
& \left.-\frac{1}{18}N_{,X\bar{X}}b_m b^m\right)\left(8X\bar{X}+\frac{1}{9}b_m b^m\right)+6X\left(\overline{\mathcal{F}}+\overline{\Omega}\right)+6\bar{X}\left(\mathcal{F}+\Omega\right) \\
& +12X\bar{X}\left(N+J\right)-J^{-1}_{,\phi\bar{\phi}}\left|2XJ_{,\phi}+\Omega_{,\phi}\right|^2.
\end{aligned}
$$

(72)

When ignoring also the pseudo-vector field, $b_m = 0$, the Lagrangian above can be rewritten to the form

如果再忽略赝矢量场 $b_m = 0$ ，上述拉格朗日量可以改写为如下形式

$$
e^{-1}\mathcal{L}_{\text{bos.}} = \frac{A}{2}R+\frac{B}{12M^2}R^2-\frac{12B}{M^2}\partial X\partial\bar{X}-J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi}-U, \tag{73}
$$

with the specific scalar potential U , where we have defined

其中具有特定标量势 U ，我们定义

$$
A \equiv -\frac{1}{6}\left(\mathcal{F}_{,X}+\overline{\mathcal{F}}_{,\bar{X}}+2N_{,X}X+2N_{,\bar{X}}\bar{X}-8N_{,X\bar{X}}X\bar{X}+2N+2J\right), \tag{74}
$$

$$
B \equiv \frac{1}{12}M^2 N_{,X\bar{X}} \tag{75}
$$

$$
\begin{aligned}
U \equiv\ & 4X\bar{X}\left(\mathcal{F}_{,X}+\overline{\mathcal{F}}_{,\bar{X}}+2N_{,X}X+2N_{,\bar{X}}\bar{X}-4N_{,X\bar{X}}X\bar{X}-N-J\right) \\
& -6X\left(\overline{\mathcal{F}}+\overline{\Omega}\right)-6\bar{X}\left(\mathcal{F}+\Omega\right)+J^{-1}_{,\phi\bar{\phi}}\left|2XJ_{,\phi}+\Omega_{,\phi}\right|^2.
\end{aligned} \tag{76}
$$

As can be seen from Eq. (76) when $X = 0$, a Minkowski vacuum requires $\Omega_{,\phi} = 0$. This leads to $F = 0$ from Eq. (71). Hence, in order to break SUSY in a Minkowski vacuum, one needs a nonvanishing vacuum expectation value (VEV) of $X, \langle X\rangle \neq 0$. If at the onset of inflation the fields ϕ and X are stabilized around zero, as is the case in the models under consideration, there will be a nontrivial multi-field dynamics at smaller (than CMB) scales when the Jordan frame potential $U\left(X, \bar{X}, \phi, \overline{\phi}\right)$ starts to control the dynamics (assuming it is initially suppressed), and the inflationary trajectory turns toward the nonvanishing VEV of X and ϕ. Therefore, the canonical Kähler potential and the linear superpotential of the standard Polonyi model have to be generalized for spontaneous SUSY breaking in the modified supergravity.

从式 (76) 可以看出，当 $X = 0$ 时，闵氏真空要求 $\Omega_{,\phi} = 0$。结合式 (71) 可得 $F = 0$。因此，要在闵氏真空中破缺超对称，需要 $X, \langle X\rangle \neq 0$ 具有非零真空期望值 (VEV)。如果在暴胀开始时场 ϕ 和 X 稳定在零附近，这正是本文研究模型的情况，当约旦架势 $U\left(X, \bar{X}, \phi, \overline{\phi}\right)$ 开始主导动力学 (假设其初始被压低) 时，小于 CMB 的尺度上会存在非平庸的多场动力学，暴胀轨迹会向 X 和 ϕ 的非零真空期望值偏移。因此，必须推广标准波洛尼模型的正则凯勒势和线性超势，才能在改进超引力中实现自发超对称破缺。

The Lagrangian (73) in the Jordan frame can be transformed to the dual (scalar-tensor) Lagrangian with the scalaron field φ in the Einstein frame. One finds [75]

约旦架中的拉格朗日量 (73) 可以变换为爱因斯坦架中带标量子场 φ 的对偶 (标量-张量) 拉格朗日量，可得 [75]

(77)

$$e^{-1}\mathcal{L}_{\text{bos.}} = \frac{1}{2}R - \frac{1}{2}\partial\varphi\partial\varphi - e^{-\sqrt{\frac{2}{3}}\varphi}\left(\frac{12B}{M^2}\partial X\partial\bar{X} + J_{,\phi\bar{\phi}}\partial\phi\partial\overline{\phi}\right)$$

$$-\frac{3M^2}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U.$$

The potentials in the minimalistic (Starobinsky supergravity) setup are given by

极简 (斯塔罗宾斯基超引力) 框架下的各势由下式给出

$$\mathcal{F} = -3X,\ N = \frac{12}{M^2}X\bar{X} - \frac{72}{M^4}\zeta\left(X\bar{X}\right)^2, \tag{78}$$

just needed for the proper embedding of the Starobinsky $\left(R + R^2\right)$ gravity model of inflation into the modified supergravity; see section "Starobinsky-Type Supergravity and PBH Production." In particular, the parameter M is proportional to the scalaron mass m_φ as $m_\varphi^2 = M^2/\langle B\rangle$ after assuming that $\left\langle Ae^{-\sqrt{2/3}\varphi}\right\rangle = 1$ and $\langle U\rangle = 0$, where the angle brackets denote the vacuum expectation values (VEV). The extra (second) term in N with the real parameter $\zeta > 0$ is needed for stabilization of the inflationary trajectory and the vacuum [44].

这只是为了将 Starobinsky $\left(R + R^2\right)$ 暴胀引力模型正确嵌入修正超引力所必需的；参见“Starobinsky 型超引力与原初黑洞产生”一节。特别地，假设 $\left\langle Ae^{-\sqrt{2/3}\varphi}\right\rangle = 1$ 和 $\langle U\rangle = 0$ (角括号代表真空期望值 VEV) 后，参数 M 与标量子质量 m_φ 成比例，比例关系为 $m_\varphi^2 = M^2/\langle B\rangle$。$N$ 中带实参数 $\zeta > 0$ 的额外 (第二项) 项是稳定暴胀轨迹和真空所必需的 [44]。

After the rescalings

重新标度后

$$X \to MX/\sqrt{12},\ \Omega \to M\Omega/\sqrt{3}, \tag{79}$$

the Lagrangian takes the form

拉格朗日量形式为

(80)

$$e^{-1}\mathcal{L}_{\text{bos.}} = \frac{1}{2}R - \frac{1}{2}\partial\varphi\partial\varphi - e^{-\sqrt{\frac{2}{3}}\varphi}\left(B\partial X\partial\bar{X} + J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi}\right)$$

$$-\frac{3M^2}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U.$$

where the functions A, B , and U read

其中函数 A, B 和 U 为

$$A = 1 + \frac{1}{3}\left(X\bar{X} - J\right) - \frac{11}{6}\zeta\left(X\bar{X}\right)^2,\ B = 1 - 2\zeta X\bar{X},$$

$$U = M^2\left[X\bar{X}\left(1 - \frac{1}{3}J\right) - \frac{1}{3}\left(X\bar{X}\right)^2 + \frac{3}{2}\zeta\left(X\bar{X}\right)^3 - X\overline{\Omega} - \bar{X}\Omega + \frac{1}{3}J^{-1}_{,\phi\bar{\phi}}\left|XJ_{\phi} + \Omega_{,\phi}\right|^2\right].$$

(81)

The proper extension of the Polonyi model is given by [75]

Polonyi 模型的恰当推广由文献 [75] 给出

$$J = \phi\bar{\phi} - \frac{\lambda}{2}\left(\phi\bar{\phi}\right)^2, \tag{82}$$

$$\Omega = b\phi + \frac{c}{2}\phi^2 + \frac{f}{3}\phi^3, \tag{83}$$

with four real parameters (λ, b, c, f) . The Lagrangian (80) in this case reads

含四个实参数 (λ, b, c, f) 。这种情况下拉格朗日量 (80) 为

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\varphi\partial\varphi - e^{-\sqrt{\frac{2}{3}}\varphi}\left(B\partial X\partial\bar{X} + J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi}\right) - V \tag{84}$$

with the scalar potential in the Einstein frame as

爱因斯坦框架下的标量势为

$$V = \frac{3M^2}{4B}(1 - Ay)^2 + y^2U, \tag{85}$$

where we have introduced the notation

其中我们引入了记号

$$y \equiv e^{-\sqrt{2/3}\varphi} \tag{86}$$

A, B , and U are the functions of $X, \bar{X}$ and $\phi, \overline{\phi}$ with

A, B 和 U 是 $X, \bar{X}$ 和 $\phi, \overline{\phi}$ 的函数，满足

$$X = \frac{1}{\sqrt{2}}(\sigma + i\hat{\sigma}),\ \phi = \frac{1}{\sqrt{2}}(\rho + i\hat{\rho}), \tag{87}$$

where the hats are used to denote the imaginary parts (pseudo-scalars).

带帽子的符号用来表示虚部 (赝标量)。

This model can accommodate the power spectrum enhancement (peak) leading to PBH as the whole DM or as a significant part of DM for the tuned values of the parameters given in Table 2.

对于表 2 给出的调谐参数值，该模型可以实现功率谱增强 (峰)，从而形成构成全部暗物质或暗物质重要组成部分的原初黑洞。

Table 2 The parameters c, f , and λ for three selected values of b and $\Delta N_2 = 20$ and the corresponding predictions for the inflation tilts. The scalaron mass M is in Planck units

表 2 三个选定 b 和 $\Delta N_2 = 20$ 对应的参数 c, f 和 λ ，以及暴胀斜率的相关预言。标量子质量 M 以普朗克单位给出

b	C	f	λ	n_s	$r_{\max}$	M
0.01	1.8×10^{-9}	-0.007098	0.42	0.9464	0.0081	1.87×10^{-5}
0.1	1.7×10^{-7}	-0.03863	0.27	0.9463	0.0082	1.88×10^{-5}
1	1.4×10^{-5}	-0.1717	0.19	0.9434	0.0092	2.00×10^{-5}

Table 3 The SUSY breaking VEV of the auxiliary F -fields, the gravitino masses $m_{3/2}$, and the masses of pseudo-scalars $\hat{\sigma}$ and $\hat{\rho}$ for the three values of the parameter b

表 3 参数 b 取三个值时，辅助 F 场的超对称破缺真空期望值、引力微子质量 $m_{3/2}$ ，以及赝标量 $\hat{\sigma}$ 和 $\hat{\rho}$ 的质量

b	$\frac{\lvert\langle F^T\rangle\rvert}{MM_{\text{Pl}}}$	$\frac{\lvert\langle F^S\rangle\rvert}{MM_{\text{Pl}}}$	$\frac{\lvert\langle F^\phi\rangle\rvert}{MM_{\text{Pl}}}$	$\frac{\langle m_{3/2}\rangle}{M}$	$\frac{m_{\hat{\sigma}}}{M}$	$\frac{m_{\hat{\rho}}}{M}$
1	0.11	0.624	1.631	1.121	0.25	1.21
0.1	6×10^{-5}	0.048	0.155	0.092	0.77	0.19
0.01	3×10^{-8}	0.048	0.014	0.007	0.82	0.02

Table 4 The parameters, the inflation observables, and the PBH masses in three examples A, B, and C. The mass parameter varies from $M \approx 1.9 \times 10^{-5} M_{\text{Pl}}$ to $2 \times 10^{-5} M_{\text{Pl}}$

表 4 三个例子 A、B、C 中的参数、暴胀观测量和原初黑洞质量。质量参数从 $M \approx 1.9 \times 10^{-5} M_{\text{Pl}}$ 变化到 $2 \times 10^{-5} M_{\text{Pl}}$

set	b	C	f	λ	n_s	r_{max}	ΔN_2	$\widehat{f}_{\text{tot}}$	M_{PBH}
A	1	7.25×10^{-6}	-0.32037	0.251	0.9460	0.0084	18.27	1	10^{18} g
B	0.3	6.88×10^{-7}	-0.09812	0.2442	0.9460	0.0082	20.15	0.54	10^{18} g
C	0.1	7.66×10^{-8}	-0.03462	0.252	0.9434	0.0090	21.92	1	10^{18-20} g

The corresponding numerical results for the F -fields and the gravitino masses $m_{3/2}$ in the Minkowski vacuum are shown in Table 3, together with the masses of the pseudo-scalars $\hat{\sigma}$ and $\hat{\rho}$, demonstrating that the latter are not destabilized after inflation.

闵氏真空下 F 场和引力微子质量 $m_{3/2}$ 的对应数值结果，连同赝标量 $\hat{\sigma}$ 和 $\hat{\rho}$ 的质量一同展示在表 3 中，结果表明暴胀后赝标量并未失稳。

As regards a numerical calculation of the PBH masses and PBH-to-DM density fraction by using the Press-Schechter formalism, the results are illustrated by Table 4.

关于利用 Press-Schechter 形式化对原初黑洞质量和原初黑洞占暗物质密度占比进行的数值计算, 结果由表 4 给出。

The corresponding PBH fractions are given in Fig. 4 for the parameter sets A (the blue curve in Fig. 4) and B (the orange curve in Fig. 4). The set A produces $\widehat{f}_{\text{tot}} = 1$, with n_s on the margin of the 3σ CMB constraint, whereas the set B leads to $\widehat{f}_{\text{tot}} = 0.54$ with the same value of n_s . The parameter set C is excluded by 3σ .

对应的原初黑洞占比已在图 4 的参数集 A(图 4 蓝色曲线) 和参数集 B(图 4 橙色曲线) 中给出。参数集 A 产生 $\widehat{f}_{\text{tot}} = 1$, 其中 n_s 处于 3σ CMB 限制的边缘, 而参数集 B 在相同 n_s 取值下得到 $\widehat{f}_{\text{tot}} = 0.54$ 。参数集 C 被 3σ 排除。

In this supergravity model, the scalaron φ is the driver of the first stage of inflation where the CMB scale exits the horizon, whereas the second stage of inflation is driven by a combination of σ (the real part of X) and ρ (the real part of the chiral matter scalar ϕ). The beginning of the second inflationary stage gives rise to an enhancement (peak) of the power spectrum. In order to achieve the required enhancement of $O\left(10^7\right)$ in the power spectrum of scalar perturbations for a substantial PBH production, the additional term proportional to $|\phi|^4$ is needed in the Kähler potential.

在该超引力模型中，标量子 φ 是第一阶段暴胀的驱动者，CMB 尺度在此阶段退出视界，而第二阶段暴胀由 σ (X 的实部) 与 ρ (手征物质标量 ϕ 的实部) 共同驱动。第二暴胀阶段的开端会引发功率谱的增强 (峰值)。为了实现标量微扰功率谱所需的 $O\left(10^7\right)$ 增强，从而产生大量原初黑洞，需要在凯勒势中引入与 $|\phi|^4$ 成正比的附加项。

SUSY is spontaneously broken in the vacuum after inflation, while one of the parameters can be fixed to achieve the vanishing cosmological constant. The masses of the pseudo-scalars $\hat{\sigma}$ and $\hat{\rho}$ (from X and ϕ , respectively) around the vacuum are close to the inflationary Hubble scale. During inflation, the scalars $\hat{\sigma}$ and $\hat{\rho}$ are stable and have larger effective masses. There is also another real scalar (sinflaton) that also has a large (not tachyonic) effective mass during and after inflation. In the initial higher-curvature formulation of this supergravity model, the extra scalar is related to the divergence $D_m b^m$ of the vector field b_m belonging to the old-minimal supergravity multiplet. There are six real scalars in total, with three of them being stabilized and the other three being participating in inflation.

暴胀后真空会自发破缺超对称，我们可以固定其中一个参数得到为零的宇宙学常数。赝标量 $\hat{\sigma}$ 和 $\hat{\rho}$ (分别来自 X 和 ϕ) 在真空附近的质量接近暴胀哈勃尺度。暴胀过程中，标量 $\hat{\sigma}$ 和 $\hat{\rho}$ 稳定且有效质量更大。模型中还存在另一个实标量 (暴胀子 sinflaton)，它在暴胀前后都具有较大的非快子有效质量。在该超引力模型最初的高曲率表述中，这个额外标量属于旧最小超引力多重态的矢量场 b_m ，与矢量场的散度 $D_m b^m$ 相关。模型共包含 6 个实标量，其中 3 个被稳定，另外 3 个参与暴胀过程。

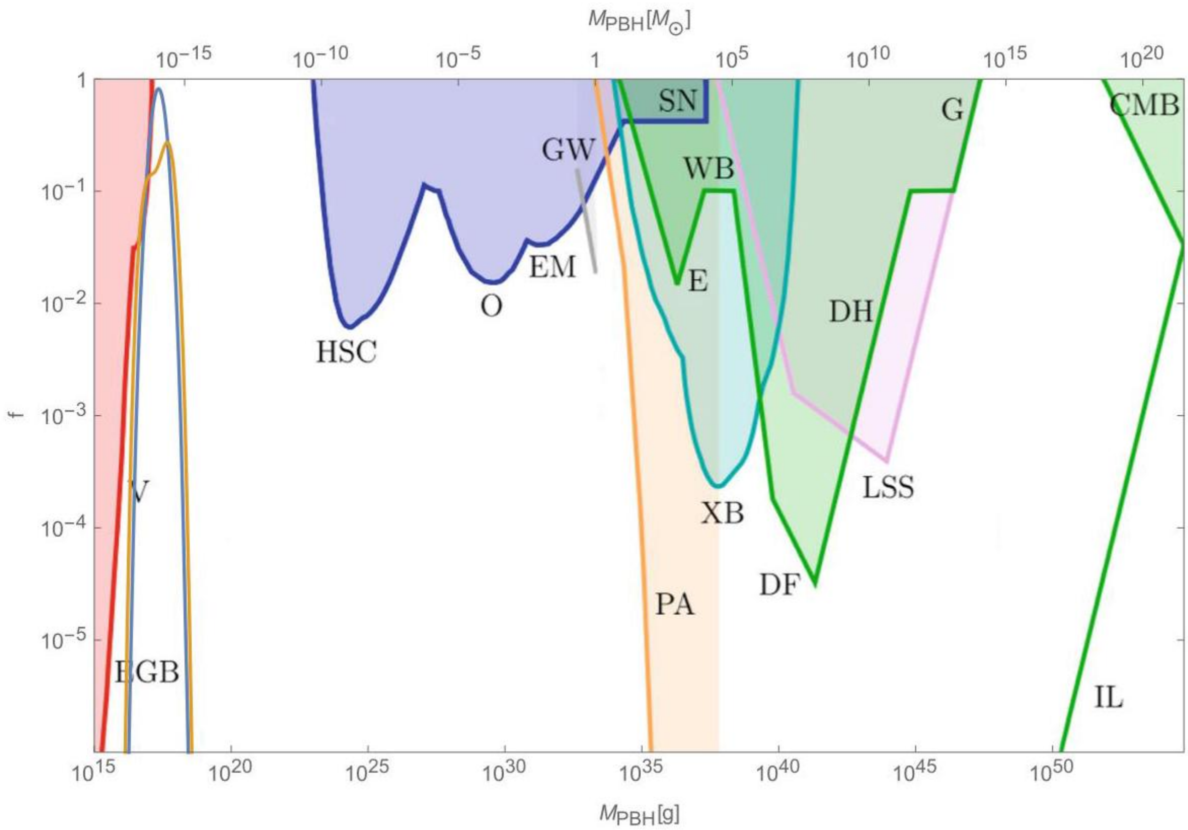

Fig. 4 The PBH-to-DM fraction for the parameter sets A (the blue curve) and B (the orange curve) in Table 4. The background of observational constraints is taken from Ref. [12]: evaporation (red), lensing (blue), gravitational waves (gray), various dynamical effects (green), accretion (light blue), large-scale structure (pink), and CMB distortions (orange)

图 4 表 4 中参数集 A(蓝色曲线) 和参数集 B(橙色曲线) 对应的原初黑洞占暗物质的比例。观测限制的背景来自文献 [12]: 蒸发 (红色)、引力透镜 (蓝色)、引力波 (灰色)、各类动力学效应 (绿色)、吸积 (浅蓝色)、大尺度结构 (粉色) 与 CMB 畸变 (橙色)

The produced PBH may describe the whole DM or its significant fraction. Current observational constraints on PBH allow only a limited mass range for the whole PBH-DM between 10^{16} g and 10^{21} g . The spectral tilt n_s is rather low, though it is within the 3σ CMB constraint when the parameter b (a free parameter in our model) is not smaller than $O\left(10^{-1}\right)$.

产生的原初黑洞可以解释全部暗物质或其中的相当一部分。当前对原初黑洞的观测限制仅允许全部由原初黑洞构成暗物质的情况存在一个介于 10^{16} g 和 10^{21} g 之间的有限质量范围。谱倾角 n_s 相当小，但当参数 b (我们模型中的自由参数) 不小于 $O\left(10^{-1}\right)$ 时，它仍满足 3σ CMB 限制。

The scale of SUSY breaking is high, about $\mathcal{O}\left(10^{13}\right)$ GeV, being under the GUT and inflation scales, which is reflected in the gravitino masses shown in Table 3.

超对称破缺的尺度很高，约为 $\mathcal{O}\left(10^{13}\right)$ GeV，低于大统一尺度与暴胀尺度，这一点反映在表 3 给出的引力微子质量中。

The second-order GW background induced by the enhanced scalar perturbations was numerically computed in Ref. [75] confirming those GW may be accessible by the future space-based GW experiments, as is expected from the (low-mass) PBH-DM scenarios [66]. For instance, in the case of LISA, the induced GW frequency should not be far away from 3.4mHz , which implies the PBH masses of the order $10^{-12}M_{\odot} \sim 10^{21}$ g [66] that is close to the PBH masses one gets from the modified supergravity models.

由增强的标量微扰诱导的二阶引力波背景已在文献 [75] 中通过数值计算得到，结果证实正如低质量原初黑洞暗物质场景 [66] 所预期的，这些引力波可被未来的天基引力波实验探测到。例如，对于 LISA 来说, 诱导引力波的频率应与 3.4mHz 相差不大, 这意味着原初黑洞质量约为 $10^{-12}M_{\odot} \sim 10^{21}$ g [66]，与修改超引力模型得到的原初黑洞质量接近。

Gravitino DM Genesis

引力微子暗物质起源

In the preceding sections, all fermionic fields were ignored in cosmological applications of supergravity. However, they can also play the important role in cosmology. The fundamental consequence of supergravity is the existence of the spin-3/2 particle called gravitino that is the superpartner of graviton with vanishing charges. After SUSY breaking via super-Higgs mechanism, gravitino particles get masses and can be viable DM candidates called SGIMP (supergravitationally interacting massive particles). Their stability (or metastability) can be secured by demanding preservation of the R-parity, when gravitino is identified with the lightest SUSY particle (LSP). In this section, we introduce the Polonyi-Starobinsky (PS) supergravity [14] and describe production of Polonyi and gravitino particles by Schwinger effect at the scales close to the scale of Starobinsky inflation. The Polonyi mass is slightly higher two gravitino masses, so that Polonyi particles are unstable and decay into two gravitinos. Part of (or whole) cold DM composed of gravitinos can be achieved. This DM production channel may be complementary to the PBH production and implies the composite nature of DM built from massive gravitinos and PBH. In this scenario, the parameter space of the inflaton potential is directly related to the dark matter one, providing a new unifying framework of inflation and dark matter genesis. Due to the superheavy masses of gravitinos, no constraints from the Big Bang nucleosynthesis (BBN) arise, while the gravitino overproduction problem can also be avoided. This framework can be embedded into the ”flipped” SUSY GUT theories inspired by heterotic string compactifications on Calabi-Yau threefolds [76], thus unifying particle physics with quantum gravity.

在之前的章节中，我们在超引力的宇宙学应用中忽略了所有费米子场。但它们在宇宙学中也能发挥重要作用。超引力的一个基本结论是存在一种名为引力子的自旋 3/2 粒子，它是引力子的超对称伙伴，电荷为零。经由超希格斯机制发生自发超对称破缺后，引力微子粒子获得质量，可成为可行的暗物质候选者，被称为 SGIMP(超引力相互作用大质量粒子)。当引力微子是最轻超对称粒子 (LSP) 时，要求 R 宇称守恒即可保证其稳定性 (或亚稳定性)。在本节中，我们介绍了 Polonyi-Starobinsky(PS) 超引力 [14]，描述了在接近 Starobinsky 暴胀的能标下，施温格效应对 Polonyi 粒子和引力微子的产生过程。Polonyi 粒子的质量略高于两倍引力微子质量，因此 Polonyi 粒子不稳定，会衰变为两个引力微子。由此可以形成部分 (或全部) 由引力微子构成的冷暗物质。这种暗物质产生通道可以与原初黑洞产生过程互补，说明暗物质是由大质量引力微子和原初黑洞共同构成的复合体系。在该情景下，暴胀子势的参数空间与暗物质的参数空间直接相关，为暴胀与暗物质起源提供了一个全新的统一框架。由于引力微子质量超重，该理论不受大爆炸核合成 (BBN) 的限制，同时也可以避免引力微子过度产生的问题。该框架可以嵌入受 Calabi-Yau 三维流形上杂化弦紧化启发得到的“翻转”超对称大统一理论中 [76]，从而实现粒子物理与量子引力的统一。

Polonyi-Starobinsky (PS) Supergravity

波洛尼-斯塔罗宾斯基 (PS) 超引力

The PS supergravity is obtained by combining chiral matter superfields described by the Lagrangian (29) with the minimal supergravity described by the Lagrangian (36), with Starobinsky's inflaton belonging a massive vector supermultiplet. The Lagrangian is given by

PS 超引力由拉格朗日量 (29) 描述的手征物质超场与拉格朗日量 (36) 描述的最小超引力结合得到，其中斯塔罗宾斯基暴胀子属于有质量矢量超多重态。其拉格朗日量为

$$\mathcal{L} = \int d^2\theta 2\mathcal{E}\left\{\frac{3}{8}\left(\overline{\mathcal{D}\mathcal{D}} - 8\mathcal{R}\right)e^{-\frac{1}{3}(K+2J)} + \frac{1}{4}W^{\alpha}W_{\alpha} + \mathcal{W}\right\} + \text{ h.c.} \tag{88}$$

After eliminating the auxiliary fields, one finds the bosonic part of the Lagrangian (88) as [47]

消去辅助场后，可得拉格朗日量 (88) 的玻色子部分为 [47]

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - K_{A\bar{A}}\partial_m A\partial^m\bar{A} - \frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}J''\partial_m C\partial^m C - \frac{1}{2}J''B_m B^m - \mathcal{V},$$

(89)

where the capital Latin subscripts denote the derivatives with respect to A ands $\bar{A}$ and the primes denote the derivatives with respect to C . The scalar potential reads

其中大写拉丁下标表示对 A 和 $\bar{A}$ 的导数，撇号表示对 C 的导数。标量势为

$$\mathcal{V} = \frac{g^2}{2}J'^2 + e^{K+2J}\left\{K^{-1}_{A\bar{A}}\left(\mathcal{W}_A + K_A\mathcal{W}\right)\left(\overline{\mathcal{W}}_{\bar{A}} + K_{\bar{A}}\overline{\mathcal{W}}\right) - \left(3 - 2\frac{J'^2}{J''}\right)\mathcal{W}\overline{\mathcal{W}}\right\},$$

(90)
generalizing the standard formula (33).

推广了标准公式 (33)。

The bosonic field components of the supergravity superfields are

超引力超场的玻色场分量为

$$2\mathcal{E}\,|= e,\ \mathcal{D}\mathcal{D}\,(2\mathcal{E})| = 4e\bar{M}, \tag{91}$$

$$\mathcal{R}\,\Big|= -\frac{1}{6}M,\ \mathcal{D}\mathcal{D}\mathcal{R}\Big| = -\frac{1}{3}R + \frac{4}{9}M\bar{M} + \frac{2}{9}b_m b^m - \frac{2}{3}i\mathcal{D}_m b^m,$$

in terms of the vierbein determinant $e \equiv \det e_m^a$, the space-time scalar curvature R , and the auxiliary fields given by the complex scalar M , and the real pseudo-vector b_m . The superspace covariant derivatives are used to define the field components here, unlike section "Supergravity and Inflation."

用 vierbein 行列式 $e \equiv \det e_m^a$ 、时空标量曲率 R 、复标量给出的辅助场 M 以及实赝矢量 b_m 表示。此处用超空间协变导数定义场分量，与“超引力与暴胀”一节不同。

The bosonic field components of the chiral superfields are defined by

手征超场的玻色场分量定义为

$$\Phi\,|= A,\ \mathcal{D}_\alpha\mathcal{D}_\beta\Phi| = -2\varepsilon_{\alpha\beta}F,\ \overline{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\Phi\,|= -2i\sigma_{\alpha\dot\alpha}{}^m\partial_m A, \tag{92}$$

$$\overline{\mathcal{D}\mathcal{D}}\mathcal{D}\mathcal{D}\Phi\,|= 16\Box A + \frac{32}{3}ib_a\partial^a A + \frac{32}{3}FM,$$

$$V\,|= C,\ \mathcal{D}_\alpha\mathcal{D}_\beta V| = \varepsilon_{\alpha\beta}X,\ \overline{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha V\,|= \sigma_{\alpha\dot\alpha}{}^m\,(B_m - i\partial_m C)\,,$$

$$\mathcal{D}_\alpha W^\beta\,|\equiv -\frac{1}{4}\mathcal{D}_\alpha\left(\overline{\mathcal{D}\mathcal{D}} - 8\mathcal{R}\right)\mathcal{D}^\beta V = \frac{1}{2}\sigma_{\alpha\dot\alpha}{}^m\bar{\sigma}^{\dot\alpha\beta n}\,(\mathcal{D}_m\partial_n C + iF_{mn}) + \delta_\alpha{}^\beta\left(D + \frac{1}{2}\Box C\right),$$

$$\overline{\mathcal{D}\mathcal{D}}\mathcal{D}\mathcal{D}V\,|= \frac{16}{3}b^m\,(B_m - i\partial_m C) + 8\Box C - \frac{16}{3}MX + 8D,$$

in terms of the physical fields (A, C, B_m) , the auxiliary fields (F, X, D) , and the vector field strength $F_{mn} = \mathcal{D}_m B_n - \mathcal{D}_n B_m$.

用物理场 (A, C, B_m) 、辅助场 (F, X, D) 和矢量场强 $F_{mn} = \mathcal{D}_m B_n - \mathcal{D}_n B_m$ 表示。

As is clear from Eq. (89), the absence of ghosts requires $J''(C) > 0$. The Kähler potential and the superpotential of the standard Polonyi model are

由式 (89) 可知，无鬼要求满足 $J''(C) > 0$ 。标准波洛尼模型的凯勒势和超势为

$$K = \Phi\overline{\Phi}, \;\; \mathcal{W} = \mu\,(\Phi + \beta)\,, \tag{93}$$

with the parameters μ and β . Unlike section ”Volkov-Akulov-Starobinsky Super-gravity,” no nilpotency condition is imposed on the chiral superfield Φ . The model includes the single-field (C) inflationary model, whose D -type scalar potential is given by

其中参数为 μ 和 β 。与“沃尔科夫-阿库洛夫-斯塔罗宾斯基超引力”一节不同，本模型不对手征超场 Φ 施加幂零条件。该模型包含单场 (C) 暴胀模型，其 D 型标量势为

$$V\,(C) = \frac{g^2}{2}(J')^2. \tag{94}$$

The Minkowski vacuum conditions (after inflation) are satisfied at $J' = 0$, which implies

(暴胀后) 闵氏真空条件在 $J' = 0$ 处满足，由此可得

$$\langle A\rangle = \sqrt{3} - 1 \;\; \text{and} \;\; \beta = 2 - \sqrt{3}. \tag{95}$$

This solution describes a stable Minkowski vacuum with spontaneous SUSY breaking at arbitrary scale $\langle F\rangle = \mu$. The related gravitino mass is given by

该解描述了在任意标度 $\langle F\rangle = \mu$ 下通过自发超对称破缺得到的稳定闵氏真空。对应的引力微子质量为

$$m_{3/2} = \mu e^{2-\sqrt{3}+\langle J\rangle}. \tag{96}$$

There is also a complex (Polonyi) scalar of mass

物理谱中还存在一个质量为

$$M_A = 2\mu e^{2-\sqrt{3}} \geq 2m_{3/2} \tag{97}$$

and a massless fermion in the physical spectrum. The inequality in Eq. (97) is saturated in the original Polonyi model, but it is not the case in the model under consideration because $\langle J\rangle < 0$.

的复 (波洛尼) 标量，以及一个无质量费米子。原始波洛尼模型满足式 (97) 的等式，但本文研究的模型不满足，因为 $\langle J\rangle < 0$ 。

The D-type scalar potential associated with the Starobinsky model arises when [46]

当 [46] 满足时，会产生与斯塔罗宾斯基模型相关的 D 型标量势

$$J\,(C) = \frac{3}{2}\,(C - \ln C) \tag{98}$$

that implies

这意味着

$$J'(C) = \frac{3}{2}\left(1 - C^{-1}\right) \text{ and } J''(C) = \frac{3}{2}\left(C^{-2}\right) > 0. \tag{99}$$

According to Eq. (89), the inflaton field ϕ with the canonical kinetic term is related to the field C by the field redefinition

根据式 (89)，具有正则动能项的暴胀子场 ϕ 通过场重定义与场 C 联系起来

$$C = \exp\left(\sqrt{2/3}\phi\right) \tag{100}$$

Thus, one gets the scalar potential of the Starobinsky model,

因此，我们得到 Starobinsky 模型的标量势，

$$V_{\text{Star.}}(\phi) = \frac{9g^2}{8}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2. \tag{101}$$

In order to break SUSY spontaneously, adding the standard Fayet-Iliopoulos (FI) term does not work because it leads to the gauged R-symmetry [77] and, hence, to highly restricted superpotentials. However, there exists the alternative FI term, without gauging the R-symmetry. It reads [78]

为了实现自发超对称破缺，引入标准的费耶特-伊里奥普洛斯 (FI) 项是行不通的，因为它会得到定规 R 对称性 [77]，进而对超势带来极强的限制。不过，存在一种不对 R 对称性定规的替代 FI 项，其形式为 [78]

$$\mathcal{L}_{\text{FI}} = 8\xi \int d^4\theta E \frac{W^2\bar{W}^2}{\mathcal{D}^2 W^2 \overline{\mathcal{D}}^2 \bar{W}^2} \mathcal{D}^\alpha W_\alpha \tag{102}$$

with the constant real parameter ξ. This term is manifestly SUSY- and gauge-invariant and does not include the higher derivatives of the field components; however, it has the inverse powers of the auxiliary field D (up to the forth order) in the fermionic sector only. When all fermions are ignored, the scalar D enters the bosonic action as a quadratic polynomial. The Kähler-Weyl gauge invariance is broken by the FI term (102) but can be restored by further modifications [49,50].

其中 ξ 是常数实参数。该项明显满足超对称不变性和规范不变性，不包含场分量的高阶导数；但它仅在费米子区包含辅助场 D 的负幂次 (最高到四阶)。当忽略所有费米子时，标量 D 以二次多项式的形式出现在玻色作用量中。Kähler-Weyl 规范不变性被 FI 项 (102) 破坏，但可以通过进一步修正恢复 [49,50]。

With the J -function and the FI term (102), the scalar potentials get modified as

引入 J 函数和 FI 项 (102) 后，标量势修正为

$$V_D = \frac{g^2}{2}\left[J' + \xi e^{\frac{1}{3}(K+2J)}\right]^2, \tag{103}$$

$$V_F = \mu^2 e^{\bar{A}A+2J}\left\{\left|\bar{A}A + A\beta + 1\right|^2 - \left(3 - 2\frac{J'^2}{J''}\right)|A+\beta|^2\right\}. \tag{104}$$

Demanding the V_D to reproduce the Starobinsky potential yields the first-order nonlinear differential equation

要求 V_D 重现 Starobinsky 势，可得到如下一阶非线性微分方程

$$\frac{dJ}{dC} + \xi e^{\frac{1}{3}(K+2J)} = -\frac{3}{2}\left(1 + \frac{1}{C}\right). \tag{105}$$

Since the Polonyi field A should stay in its minimum at $A = \langle A \rangle$, one introduces the effective (field-dependent) alternative FI term $\tilde{\xi}\left(A, \bar{A}\right) = \xi e^{\frac{1}{3}K(A,\bar{A})}$ together with its VEV, $\tilde{\xi} = \xi e^{\frac{1}{3}K(\langle A \rangle, \langle \bar{A} \rangle)}$. Then, Eq. (105) takes the form

由于 Polonyi 场 A 需要稳定在 $A = \langle A \rangle$ 处的极小值，我们引入依赖场的有效替代 FI 项 $\tilde{\xi}\left(A, \bar{A}\right) = \xi e^{\frac{1}{3}K(A,\bar{A})}$ 及其真空期望值 $\tilde{\xi} = \xi e^{\frac{1}{3}K(\langle A \rangle, \langle \bar{A} \rangle)}$ ，此时式 (105) 可写为

$$\frac{dJ}{dC} + \tilde{\xi} e^{\frac{2}{3}J} = -\frac{3}{2}\left(1 + \frac{1}{C}\right). \tag{106}$$

Without the FI term, $\tilde{\xi} = 0$, one finds the asymptotic behavior $J \sim -\frac{3}{2}C > 0$ for large negative C , which causes an instability. The instability can be removed when the function J would approach a constant instead, because large negative values of C exactly correspond to a plateau (slow roll) of Starobinsky inflation. Indeed, Eq. (106) can be easily integrated for $\left|C^{-1}\right| \ll 1$, with the result

当不存在 FI 项 $\tilde{\xi} = 0$ 时，我们得到大负 C 下的渐近行为 $J \sim -\frac{3}{2}C > 0$ ，这会引发不稳定性。若函数 J 趋近于常数，不稳定性即可消除，因为大负 C 恰好对应 Starobinsky 暴胀的平台 (慢滚) 阶段。事实上，当 $\left|C^{-1}\right| \ll 1$ 时式 (106) 很容易积分，结果为

$$J(C) \approx J_\infty - \frac{3}{2}\ln\left(1 - e^{C-C_0}\right), \tag{107}$$

where C_0 is the integration constant, and we have used the notation

其中 C_0 是积分常数，我们采用了如下记号

$$J_\infty = \frac{3}{2}\ln\left(\frac{3}{-2\tilde{\xi}}\right). \tag{108}$$

As is clear from Eqs. (107) and (108), demanding

从式 (107) 和 (108) 可以明显看出，要求

$$\xi < 0 \tag{109}$$

implies $\tilde{\xi} < 0$ also, whereas the function J fast approaches the constant J_∞ from above, with C taking large negative values. It is worth noticing that $J_\infty = 0$ at the "critical" value $\tilde{\xi} = -3/2$.

意味着 $\tilde{\xi} < 0$ 也成立，同时当 C 取大负值时，函数 J 快速从上方趋近于常数 J_∞ 。值得注意的是，$J_\infty = 0$ 在“临界”值 $\tilde{\xi} = -3/2$ 处满足该性质。

The FI-modified inflationary scalar potential of PS supergravity during slow roll is

慢滚阶段，经 FI 修正的 PS 超引力暴胀标量势为

$$\mathcal{V} = \frac{9}{8}g^2 M_{\rm Pl}^4\left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)^2 + \mu^2 M_{\rm Pl}^{-2}\exp\left(M_{\rm Pl}^{-2}\bar{A}A + 2J_\infty\right)$$

$$\times\left\{\left|\bar{A}A + A\beta + M_{\rm Pl}^2\right|^2 - 3M_{\rm Pl}^2\left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)|A+\beta|^2\right\} \equiv V_D + V_F,$$

(110)

where the dependence upon the reduced Planck mass $M_{\rm Pl}$ is restored. At large values of ϕ (and fixed $\bar{A}, A$), the V_F goes to zero, while $V_D \to 9g^2 M_P^4/8$.

其中恢复了对约化普朗克质量 $M_{\rm Pl}$ 的依赖。当 ϕ 取大值 (且 $\bar{A}, A$ 固定) 时，V_F 趋近于零，而 $V_D \to 9g^2 M_P^4/8$ 。

Polonyi and Gravitino Dynamics

波洛尼场与引力微子动力学

Since the equations of motion in PS supergravity are very complicated, let us keep only the leading order with respect to the inverse Planck mass, temporarily neglect the couplings of Polonyi and gravitino particles to inflaton, and return them back in the end of this section. Then the effective action of the Polonyi field in the FLRW background, in the co-moving coordinates, reads

由于 PS 超引力的运动方程非常复杂，我们仅保留普朗克质量逆次的领头阶，暂时忽略波洛尼粒子与引力微子对胀子的耦合，并在本节末尾重新引入这些耦合。那么共动坐标下 FLRW 背景中波洛尼场的有效作用量可写为

$$I[A] = \int dt \int d^3x \frac{a^3}{2}\left(\dot{A}^2 - \frac{1}{a^2}(\nabla A)^2 - M_A^2 A^2 - \zeta R A^2\right), \quad (111)$$

where ζ is the non-minimal coupling constant of the Polonyi field to gravity, A is the Polonyi field, M_A stands for its mass, R is the Ricci scalar, and a is the FLRW scale factor.

其中 ζ 是波洛尼场与引力的非最小耦合常数，A 为波洛尼场，M_A 代表其质量，R 是里奇标量，a 是 FLRW 标度因子。

The mode decomposition of the Polonyi field reads

波洛尼场的模分解可写为

$$A(\mathbf{x}) = \int d^3k(2\pi)^{-3/2}a^{-1}(\eta)\left[b_k h_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + b_k^\dagger h_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\right], \tag{112}$$

where the conformal time coordinate η and the creation/annihilation operators $b, b^\dagger$ have been introduced, while the coefficient functions h, h^+ have been normalized as follows:

这里引入了共形时间坐标 η 和产生/湮灭算符 $b, b^\dagger$ ，而系数函数 h, h^+ 按如下方式归一化:

$$h_k h_k'^* - h_k' h_k^* = i \tag{113}$$

Because of Eqs. (111) and (112), the equations of motion for the modes are

根据式 (111) 和 (112)，各模的运动方程为

$$h_k''(\eta) + \omega_k^2(\eta)h_k(\eta) = 0, \text{ where } \omega_k^2 = 5\frac{a''}{a} + k^2 + M_A^2 a^2, \tag{114}$$

and $h'' = d^2h/d\eta^2$. Equation (114) can be conveniently rescaled by using some reference scales $a(\eta_*) \equiv a_*$ and $H(\eta_*) = H_*$ as follows:

且 $h'' = d^2h/d\eta^2$ 。利用参考标度 $a(\eta_*) \equiv a_*$ 和 $H(\eta_*) = H_*$ 可方便地对式 (114) 做标度变换，得到:

$$h_{\tilde{k}}''(\tilde{\eta}) + \left(\tilde{k}^2 + b^2\tilde{a}^2\right)h_{\tilde{k}}(\tilde{\eta}) = 0, \tag{115}$$

in terms of the rescaled quantities

用重标度后的量表示为

$$\tilde{\eta} = \eta a_* H_*, \ \tilde{a} = a/a_*, \ \tilde{k} = k/(H_* a_*).$$

The leading order of the gravitino action is given by the massive Rarita-Schwinger action,

引力微子作用量的领头阶由有质量拉里塔-施温格作用量给出,

$$I[\psi] = \int d^4xe\overline{\psi}_\sigma \mathcal{R}^\sigma\{\psi\}, \tag{116}$$

where the gravitino kinetic operator has been introduced,

这里我们已经引入了引力微子动能算符,

$$\mathcal{R}^\sigma\{\psi\} = m_{3/2}\gamma^{\sigma\upsilon}\psi_\upsilon + i\gamma^{\sigma\upsilon\rho}\mathcal{D}_\upsilon\psi_\rho, \tag{117}$$

with the supercovariant derivative

其中超协变导数为

$$\mathcal{D}_\mu\psi_\upsilon = -\Gamma^\rho_{\mu\upsilon}\psi_\rho + \partial_\mu\psi_\upsilon + \frac{1}{4}\omega_{\mu ab}\gamma^{ab}\psi_\upsilon, \tag{118}$$

in the γ -notation $\gamma^{\mu_1\cdots\mu_n} = \gamma^{[\mu_1}\dots\gamma^{\mu_n]}$ with unit weight of the antisymmetrized product.

采用 γ 记号表示 $\gamma^{\mu_1\cdots\mu_n} = \gamma^{[\mu_1}\dots\gamma^{\mu_n]}$，满足反对称积单位权。

Since the supergravity torsion is of the second order with respect to the inverse Planck mass, we can ignore it in the leading order approximation. The $\Gamma^\rho_{\mu\nu}$ can be represented by the standard symmetric Christoffel symbols that are actually cancelled from the Rarita-Schwinger action (116). The Rarita-Schwinger action leads to the gravitino equation of motion:

由于超引力挠率是普朗克质量逆次的二阶项，我们可以在领头阶近似中忽略它。$\Gamma^\rho_{\mu\nu}$ 可以用标准对称克里斯托费尔符号表示，而这些符号实际上会从拉里塔-施温格作用量 (116) 中抵消掉。由拉里塔-施温格作用量可得引力微子的运动方程:

$$\left(i\mathcal{D} - m_{3/2}\right)\psi_\mu - \left(i\mathcal{D}_\mu + \frac{m_{3/2}}{2}\gamma_\mu\right)\gamma\cdot\psi = 0. \tag{119}$$

In the flat FLRW background, Eq. (119) reduces to

在平坦 FLRW 背景下，式 (119) 约化为

$$i\gamma^{mn}\partial_m\psi_n = -\left(m_{3/2} + i\frac{a'}{a}\gamma^0\right)\gamma^m\partial_m\psi, \tag{120}$$

where

其中

$$\omega_{\mu ab} = 2\dot{a}a^{-1}e_{\mu[a}e^0_{b]},\ e^a_\mu = a(\eta)\,\delta^a_\mu,\ m_{3/2} = m_{3/2}(\eta)\,. \tag{121}$$

A solution to Eq. (120) is

式 (120) 的一个解为

$$\psi_\mu(x) = \int d^3\mathbf{p}(2\pi)^{-3}(2p_0)^{-1}\sum_\lambda\left\{e^{i\mathbf{k}\cdot\mathbf{x}}b_\mu(\eta,\lambda)\,a_{k\lambda}(\eta) + e^{-i\mathbf{k}\cdot\mathbf{x}}b^C_\mu(\eta,\lambda)\,a^\dagger_{k\lambda}(\eta)\right\}.$$

(122)

The equations of motion for the 3/2-helicity gravitino modes have the same form as in Eq. (114), namely,

3/2 螺旋度引力微子模式的运动方程与式 (114) 形式相同，即

$$b''_\mu(\eta,\lambda) + \widehat{C}(k,a)\,b'_\mu(\eta,\lambda) + \omega^2(k,a)\,b_\mu(\eta,\lambda) = 0, \tag{123}$$

where we have introduced the notation

此处我们引入了记号

$$\widehat{C}(k,a)\, b'_\mu(\eta,\lambda) = -2i\gamma^{\upsilon i} k_i \gamma_{\upsilon\eta}\partial^\eta b_\mu - 2\gamma_\upsilon \left(m_{3/2} + i\frac{a'}{a}\gamma^0\right) i\gamma^{\upsilon\eta}\partial_\eta b_\mu, \tag{124}$$

$$\omega^2(k,a)/2 = k^2 + m_{3/2}^2 + 2i\frac{a'}{a}\gamma^0 m_{3/2} - \left(\frac{a'}{a}\right)^2. \tag{125}$$

Following the standard procedure in the case of Dirac and Klein-Gordon equations, the mode equations of motion can be reformulated to

遵循狄拉克方程和克莱因-戈登方程的标准处理方法，可将模式运动方程改写为

$$P_\upsilon P^\upsilon b_\mu(\eta,\lambda) = 0, \tag{126}$$

where we have introduced the projector operator

此处我们引入了投影算符

$$P^\upsilon = i\gamma^{\upsilon\eta}\partial_\eta - \gamma^{\upsilon i}k_i - \left(m_{3/2} + i\frac{a'}{a}\gamma^0\right)\gamma^\upsilon = 0. \tag{127}$$

Equation (123) can be rescaled in the same way as Eq. (115).

式 (123) 可以用和式 (115) 相同的方式做标度变换。

The gravitino interaction with matter can be described via the effective gravitino mass $M_{3/2}$ that depends on the matter fields in the Rarita-Schwinger equation and satisfies $m_{3/2} = \langle M_{3/2}\rangle$. The effective gravitino mass is generated from the following interactions of gravitino with inflaton ϕ and Polonyi scalar $\widetilde{A}$ (having $\langle\widetilde{A}\rangle = 0$)：

引力微子与物质的相互作用可以通过有效引力微子质量 $M_{3/2}$ 描述，该质量依赖于拉里塔-施温格方程中的物质场，且满足 $m_{3/2} = \langle M_{3/2}\rangle$ 。有效引力微子质量由引力微子与暴涨子 ϕ 和坡隆尼标量场 $\widetilde{A}$ 的如下相互作用产生 (其中 $\langle\widetilde{A}\rangle = 0$)：

$$M_{3/2}(\phi,\widetilde{A}) =$$

$$\mu M_{\rm Pl}^{-1}\exp\left[\left(1/\sqrt{6}\right)M_{\rm Pl}^{-1}\phi + M_{\rm Pl}^{-2}\left(\bar{\widetilde{A}}\widetilde{A} + \alpha\bar{\widetilde{A}} + \alpha\widetilde{A} + \alpha^2\right)\right]\left(\widetilde{A} + \alpha + \beta\right),$$

(128)

where we have introduced $\alpha \equiv \langle A\rangle$, with $A = \alpha + \widetilde{A}$, and have restored the dependence on the Planck mass because of its relevance for phenomenological applications.

此处我们引入了 $\alpha \equiv \langle A\rangle$，满足 $A = \alpha + \widetilde{A}$，并恢复了对普朗克质量的依赖，因为它唯象学应用中十分重要。

Polonyi and Gravitino Production

波洛尼场与引力微子产生

The dynamics of the gravitino and Polonyi fields during inflation necessarily leads to their quantum production. The number density of produced particles is calculated by using the Bogoliubov transformation

暴胀过程中引力微子与波洛尼场的动力学必然会导致它们的量子产生。我们利用博戈留波夫变换计算了产生粒子的数密度

$$h_k^{\eta_1}(\eta) = \alpha_k h_k^{\eta_0}(\eta) + \beta_k h_k^{*\eta_0}(\eta). \tag{129}$$

This transformation is performed from the vacuum solution selected by the boundary conditions at $\eta = \eta_{in}$, corresponding to the initial time of inflation, to the final time $\eta = \eta_f$, when the particle creation process from inflation stops. In the inflationary epoch, the dynamical regime is $a'/a^2 \ll M_{\rm Pl}$ and $M_{\rm Pl} ba/k \ll 1$. This implies that one can consider the extremes as $\eta_{in} = -\infty$ and $\eta_f = +\infty$ and then perform the WKB semi-classical approximation. By assuming these boundary conditions, the energy density of the Polonyi particles produced during inflation reads

该变换是从满足 $\eta = \eta_{in}$ 处边界条件 (对应暴胀初始时刻) 的真空解，变换到暴胀粒子产生过程结束的末时刻 $\eta = \eta_f$ 。在暴胀时期，动力学 regime 为 $a'/a^2 \ll M_{\rm Pl}$ 和 $M_{\rm Pl} ba/k \ll 1$ 。这意味着我们可以将两端极限取为 $\eta_{in} = -\infty$ 和 $\eta_f = +\infty$ ，再进行 WKB 半经典近似。在该边界条件假设下，暴胀期间产生的波洛尼粒子的能量密度为

$$\rho_A(\eta) = M_A n_A(\eta) = M_A H_{\rm inf}^3 \left(\frac{1}{\tilde{a}(\eta)}\right)^3 \mathcal{P}_A, \tag{130}$$

where

其中

$$\mathcal{P}_A = \frac{1}{2\pi^2} \int_0^\infty d\tilde{k}\tilde{k}^2 |\beta_{\tilde{k}}|^2. \tag{131}$$

Similar equations are valid for massive gravitinos, with the power spectrum

有质量引力微子满足类似的方程，其功率谱为

$$\mathcal{P}_\psi = \frac{1}{2\pi^2} \int_0^\infty d\tilde{k}\tilde{k}^2 \left|b_\mu b^{C\mu}\right|. \tag{132}$$

The inflaton mass sets the characteristic energy scale for the Hubble parameter calculated at a fixed cosmological time $t \equiv t_f$,

暴胀子质量确定了固定宇宙时刻 $t \equiv t_f$ 处计算所得哈勃参数的特征能标,

$$H^2(t_f) \simeq m_\phi^2,\ \rho(t_f) \simeq m_\phi^2 M_{\rm Pl}^2.$$

The formula for the Polonyi particles (energy density and Polonyi mass) produced during inflation was proposed in Ref. [14]:

文献 [14] 给出了暴胀期间产生的波洛尼粒子的公式 (能量密度与波洛尼质量):

$$(\Omega_A h^2/\Omega_R h^2) \simeq \frac{8\pi}{3}\left(\frac{M_A}{M_{Pl}}\right)\left(\frac{T_{\rm reh}}{T_0}\right)\frac{n_A(t_f)}{M_{Pl}H^2(t_f)}, \tag{133}$$

where M_A is the Polonyi mass, $\Omega_R h^2 \simeq 4.31\times10^{-5}$ is the radiation energy density at today's temperature T_0 , and $\Omega_A h^2$ is the energy density of the produced Polonyi fields, all in units of the critical energy density. The similar estimate is valid for gravitino particles also. Equation (133) is motivated by the relation

其中 M_A 为波洛尼质量，$\Omega_R h^2 \simeq 4.31\times10^{-5}$ 是当前温度 T_0 下的辐射能量密度，$\Omega_A h^2$ 是所产生波洛尼场的能量密度，全部以临界能量密度为单位。引力微子粒子也满足类似的估计。式 (133) 由下述关系推导而来

$$\frac{\rho_A(t_0)}{\rho_R(t_0)} = \frac{\rho_A(t_{\rm reh})}{\rho_R(t_{\rm reh})}\left(\frac{T_{\rm reh}}{T_0}\right), \tag{134}$$

where ρ_A is the Polonyi energy density, ρ_R is the energy density of radiation, and $T_{\rm reh}$ and T_0 are the temperature of the universe calculated at reheating $t_{\rm reh}$ and today t_0 time scales, respectively. It is reasonable to assume that Polonyi particles are mainly produced after the de Sitter phase t_e , when the transition to the coherent oscillation phase starts. Given this assumption, the inflaton and Polonyi energy densities are red-shifted with almost the same dilution rate. Their co-scaling relations hold until the reheating epoch finishes and the radiation dominated stage begins. Then most of the energy density is converted into radiation, i.e.,

其中 ρ_A 是波洛尼能量密度，ρ_R 是辐射能量密度，$T_{\rm reh}$ 和 T_0 分别是再加热 $t_{\rm reh}$ 时刻和当前 t_0 时刻的宇宙温度。我们可以合理假设，波洛尼粒子主要产生于德西特阶段 t_e 之后，此时向相干振荡阶段的转变已经开始。基于该假设，暴胀子与波洛尼的能量密度的稀释率几乎相同。它们的共标度关系一直成立，直到再加热阶段结束、辐射主导时期开始。之后大部分能量密度转化为辐射，即

$$\rho_R \simeq \rho_c = \frac{3H^2 M_{Pl}^2}{8\pi}. \tag{135}$$

This implies the following relation:

由此可得下述关系:

$$\frac{\rho_A(t_{\rm reh})}{\rho_R(t_{\rm reh})}\left(\frac{\rho_A(t_e)}{M_{Pl}^2 H^2(t_e)}\right)^{-1} \simeq \frac{8\pi}{3}, \tag{136}$$

where $H(t_e)$ is the Hubble parameter at a fixed time $t \equiv t_e$. Equation (133) follows from Eq. (136).

其中 $H(t_e)$ 是固定时刻 $t \equiv t_e$ 的哈勃参数。式 (133) 可由式 (136) 导出。

According to Eq. (130), when relating Hubble scale, Polonyi mass, and the desiderata Polonyi energy density, there is about 8th-order-of-magnitude suppression of the energy density. The normalized power spectrum $\mathcal{P}_A$ cannot provide such suppression with our values for M_A and H_{inf} . However, it comes from the dilution factor $(\tilde{a})^{-3} = \left(a_f/a_i\right)^{-3}$ in Eq. (130).

根据式 (130)，关联哈勃标度、波洛尼质量和预期波洛尼能量密度后，能量密度存在约 8 个数量级的压低。归一化功率谱 $\mathcal{P}_A$ 在我们取的 M_A 和 H_{inf} 数值下无法提供这种压低，但该压低来自式 (130) 中的稀释因子 $(\tilde{a})^{-3} = \left(a_f/a_i\right)^{-3}$ 。

The semi-analytical estimates for Eq. (130) indicate that almost all Polonyi particles are produced in an excursion of the inflaton field around $\phi_e \equiv \phi(t_e)$ with $\Delta\phi \simeq 0.2$. The value of the dilution factor can be estimated from

对式 (130) 的半解析估计表明，几乎所有波洛尼粒子都产生于暴胀子场在 $\phi_e \equiv \phi(t_e)$ 附近、满足 $\Delta\phi \simeq 0.2$ 的涨落过程中。稀释因子的数值可由下式估计

$$\left(a\left(t_f\right)/a\left(t_i\right)\right)^{-3} = \exp\left[-24\pi\int_{\phi_f}^{\phi_i} d\phi V^{-1}(\phi)\, V_{,\phi}(\phi)\right] = \exp\left(-\Delta\Phi\right), \tag{137}$$

where we have defined $\Delta\Phi = \Phi\left(\phi\left(t_i\right)\right) - \Phi\left(\phi\left(t_f\right)\right)$, having in mind that $\phi\left(t_i\right) > \phi\left(t_f\right)$ and

其中我们定义了 $\Delta\Phi = \Phi\left(\phi\left(t_i\right)\right) - \Phi\left(\phi\left(t_f\right)\right)$ ，已知 $\phi\left(t_i\right) > \phi\left(t_f\right)$ 且

$$\Phi(\phi) = 48\pi\sqrt{2/3}e^{-\sqrt{2\phi/3}}\left(1 - e^{-\sqrt{2\phi/3}}\right)^{-1}. \tag{138}$$

After integrating over the effective particle production region $\Delta\phi$, we find $\Delta\Phi = 18.2$, i.e.,

在对有效粒子产生区域 $\Delta\phi$ 积分后，我们得到 $\Delta\Phi =$ 式 18.2，即

$$\left(a\left(t_f\right)/a\left(t_i\right)\right)^{-3} \simeq \exp\left(-18.2\right) \simeq 10^{-8}. \tag{139}$$

The $a\left(t_f\right)$ refers to a cosmological time t_f close to reheating. The cosmological time t_i when particle production effectively started is not far from the t_i because the inflaton field has an excursion of merely $\Delta\Phi = \Phi\left(t_i\right) - \Phi\left(t_f\right) \simeq 20$ that is proportional to $\Delta t = t_f - t_i$. But the relation between $a(t)$ and the cosmological time t is exponential. This is the origin of the very large exponential suppression (of the eighth order) between a_f and a_i despite the fact that the effective time of particle production is very short. From the physical point of view, particles produced during t_f are diluted with the factor exponentially larger then $a\left(t_i\right)$. These results are in agreement with the common expectations that particle production is most efficient toward the end of inflation [79].

$a\left(t_f\right)$ 指的是接近再加热的宇宙学时间 t_f 。粒子有效开始产生的宇宙学时间 t_i 距离 t_i 并不远，因为暴涨场的偏移仅为与 $\Delta t = t_f - t_i$ 成正比的 $\Delta\Phi = \Phi\left(t_i\right) - \Phi\left(t_f\right) \simeq 20$ 。但 $a(t)$ 和宇宙学时间 t 之间呈指数关系，这就是 a_f 与 a_i 之间存在八阶极大指数压制的根源——尽管粒子产生的有效时间非常短。从物理角度看，t_f 期间产生的粒子被稀释，其稀释因子指数大于 $a\left(t_i\right)$ 。这些结果符合“粒子产生在暴涨末期效率最高”的普遍预期 [79]。

Gravitino Mass

引力微子质量

To get the gravitino and Polonyi masses, we add a few more cosmological assumptions about relevant parameters of the reheating process and, in particular, about the reheating temperature $T_{\rm reh}$ in the scenario based on the Starobinsky inflation. This implies that all cosmological parameters can be fixed by specifying the e-folding number N_e that is in the range between 50 and 60, due to the CMB bounds. For a more precise estimate of the cold DM abundance, we choose $N_e = 55$. This leads to the precise set of inflation parameters as

为得到引力微子和 Polonyi 质量，我们对再加热过程的相关参数增加了若干宇宙学假设，尤其是基于 Starobinsky 暴涨的场景中的再加热温度 $T_{\rm reh}$ 。这意味着所有宇宙学参数都可以通过指定 e 折叠数 N_e 确定，受宇宙微波背景背景约束，该值处于 50 到 60 之间。为更精确估计冷暗物质丰度，我们选择 $N_e = 55$ 。由此得到的精确暴涨参数集合为

$$n_s = 0.964,\ r = 0.004,\ m_{\rm inf} = 3.2 \cdot 10^{13}{\rm GeV},$$

$$H_{\rm inf} = \pi M_{\rm Pl}\sqrt{P_g/2} = 1.4 \cdot 10^{14}{\rm GeV}. \tag{140}$$

Well below the inflation scale, the low-energy effective field theory is given by the standard model (SM) that has the effective number of d.o.f. as $g_* = 106.75$. Let us assume that all the SM particles originated from perturbative inflaton decay via the (Starobinsky) universal reheating mechanism, whose reheating temperature is known [80]:

远低于暴涨能标时，低能有效场论由标准模型 (SM) 给出，其有效自由度数量为 $g_* = 106.75$ 。我们假设所有标准模型粒子都来自 (Starobinsky) 普适再加热机制下的微扰暴涨子衰变，该机制的再加热温度已有结论 [80]:

$$T_{\rm reh} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma_{\rm tot} M_P} = 3 \cdot 10^9{\rm GeV}. \tag{141}$$

This value is in agreement with the successful leptogenesis mechanism of Ref. [81].

该值与文献 [81] 中成功的轻子生成机制一致。

On the other hand, the reheating temperature for heavy gravitinos is given by [82]

另一方面，重引力微子的再加热温度由文献 [82] 给出

$$T_{\rm reh} = 1.5 \cdot 10^8\ {\rm GeV}\left(\frac{80}{g_*}\right)^{1/4}\left(\frac{m_{3/2}}{10^{12}{\rm GeV}}\right)^{3/2}. \tag{142}$$

Combining Eqs. (141) and (142), we get the gravitino and Polonyi masses as

结合式 (141) 与 (142)，我们得到引力微子和 Polonyi 质量为

$$m_{3/2} = (7.7 \pm 0.8) \cdot 10^{12}\text{GeV} \ \text{ and } \ M_A = 2e^{-\langle J \rangle} m_{3/2} \approx 2m_{3/2}. \tag{143}$$

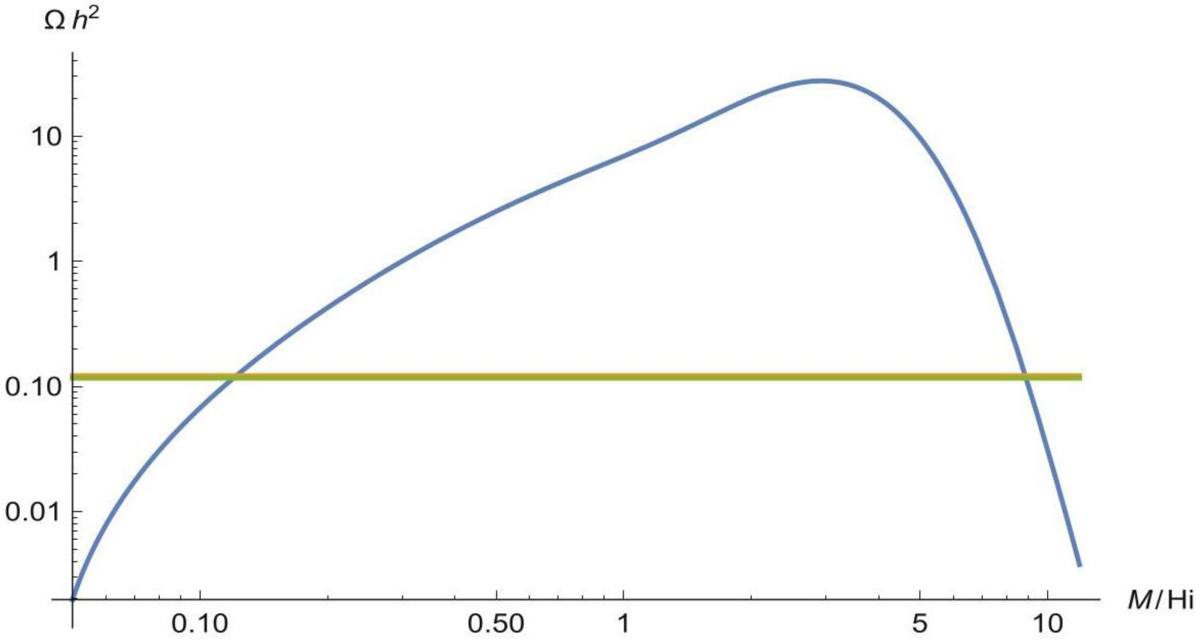

Fig. 5 Numerical simulations of the produced gravitino mass density (normalized) as a function of the Polonyi mass parameter are displayed in blue, in the parameter range compatible with inflation, reheating, and leptogenesis (at the reference point $N_e = 55$): $n_s = 0.964, r = 0.004$, $m_{\text{inf}} = 3.2 \cdot 10^{13}\text{GeV}, H_{\text{inf}} = 1.4 \cdot 10^{14}\text{GeV}$, and $T_{\text{reh}} = 3 \cdot 10^{9}\text{GeV}$. The right amount of the whole cold DM, $\Omega_{3/2}h^2 = \Omega_{DM}h^2 = 0.11$ (in orange), is generated when the Polonyi mass is $M_A \approx 2m_{3/2} = (1.54 \pm 0.2) \times 10^{13}\text{GeV}$

图 5 产生的引力微子质量密度 (归一化) 关于 Polonyi 质量参数的数值模拟以蓝色显示，参数范围与暴涨、再加热和轻子生成兼容 (参考点为 $N_e = 55$): $n_s = 0.964, r = 0.004$ 、$m_{\text{inf}} = 3.2 \cdot 10^{13}\text{GeV}, H_{\text{inf}} = 1.4 \cdot 10^{14}\text{GeV}$ 和 $T_{\text{reh}} = 3 \cdot 10^{9}\text{GeV}$ 。当 Polonyi 质量为 $M_A \approx 2m_{3/2} = (1.54 \pm 0.2) \times 10^{13}\text{GeV}$ 时，恰好产生总量正确的全部冷暗物质 $\Omega_{3/2}h^2 = \Omega_{DM}h^2 = 0.11$ (橙色)

These masses are compatible with the whole abundance of cold DM, according to the numerical estimates in Fig. 5, which lends further support toward the conjecture in Eq. (133).

根据图 5 中的数值估计，这些质量与全部冷暗物质丰度兼容，这为式 (133) 的猜想提供了进一步支持。

Decay and Annihilation Channels

衰变与湮灭道

With the cold DM made of massive gravitinos, it is important to evaluate the competition between Schwinger effect and decay channels. Polonyi particles decay into gravitino pairs with the decay rate [83]

当冷暗物质由大质量引力微子构成时，评估施温格效应与衰变道之间的竞争十分重要。波洛尼粒子衰变为引力微子对，衰变率为 [83]

$$\Gamma\left(A \rightarrow \psi_{3/2}\psi_{3/2}\right) \simeq \frac{3}{288\pi} M_A^3 / m_{3/2}^2 \simeq 2.6 \times 10^{-2} m_{3/2}. \tag{144}$$

This channel is a direct consequence of the gravitino mass generation mechanism from the nonvanishing Polonyi vacuum expectation value. Since $m_{3/2} = 7.7 \,.\, 10^{12}\mathrm{GeV}$, it implies $\Gamma \ll H_{\text{inf}}$. Hence, the decay time of Polonyi into gravitinos is much larger the production time during inflation: $\tau_{A\rightarrow\psi_{3/2}\psi_{3/2}} \gg \tau_{\text{inflation}}$. Decays of Polonyi particles into gravitinos are, therefore, negligible during inflation, while gravitino and Polonyi particles are independently generated during the inflationary epoch. After reheating, all Polonyi particles rapidly decay into gravitinos. As a result, the Polonyi number density n_S is completely converted into a contribution to the gravitino number density as $\Delta n_\Psi = 2n_A$.

该衰变道是引力微子质量由非零波洛尼真空期望值产生机制的直接结果。由于 $m_{3/2} = 7.7 \,.\, 10^{12}\mathrm{GeV}$ ，这意味着 $\Gamma \ll H_{\text{inf}}$ 。因此，inflation 期间波洛尼衰变为引力微子的时间远长于产生时间: $\tau_{A\rightarrow\psi_{3/2}\psi_{3/2}} \gg \tau_{\text{inflation}}$ 。因此，inflation 期间波洛尼粒子衰变为引力微子的过程可忽略，引力微子与波洛尼粒子在暴胀时期独立产生。再加热后，所有波洛尼粒子会快速衰变为引力微子。最终，波洛子代数密度 n_S 完全转化为引力微子数密度的贡献，即 $\Delta n_\Psi = 2n_A$ 。

The gravitino-inflaton coupling arises from the Weyl rescaling of the vierbein in the gravitino action. On the other hand, the gravitino kinetic term does not provide any contribution because of the conformal flatness of the FLRW universe. This means that the only source of the gravitino genesis is given by the effective gravitino mass term,

引力微子-暴胀子耦合来自引力微子作用量中 Vierbein 的外尔缩放。另一方面，由于 FLRW 宇宙是共形平坦的，引力微子动能项没有任何贡献。这说明引力微子产生的唯一来源是有效引力微子质量项，

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} e^{G_{\text{tot}}/2} \overline{\psi}_\mu \gamma^{\mu\upsilon} \psi_\upsilon, G_{\text{tot}} = K + \ln|W|^2 + 2J, \tag{145}$$

while the gravitino mass is the expectation value $m_{3/2} = \left\langle e^{G_{\text{tot}}/2} \right\rangle$. The perturbative decay rate of the inflaton ϕ into a pair of gravitinos is [83]

而引力微子质量就是期望值 $m_{3/2} = \left\langle e^{G_{\text{tot}}/2} \right\rangle$ 。暴胀子 ϕ 对衰变到一对引力微子的微扰衰变率为 [83]

$$\Gamma_{\phi\rightarrow\psi_{3/2}\psi_{3/2}} = \frac{|G_\phi|^2}{288\pi} \frac{m_{\text{inf}}^5}{m_{3/2}^2 M_{Pl}^2}. \tag{146}$$

In the model under consideration, the factor G_ϕ vanishes at the minimum of the inflaton scalar potential. This implies that the perturbative production of the gravitinos from the inflaton decays is suppressed. Another contribution comes from the Polonyi production due to the inflaton decays. This decay channel is kinematically allowed since inflaton is heavier than Polonyi particle in our model. The perturbative decay rate of inflaton into a pair of Polonyi scalars - see, for example, Ref. [41] for a review - reads

在我们研究的模型中，因子 G_ϕ 在暴胀子标量势的极小值处为零。这说明暴胀子衰变产生引力微子的微扰过程被压低。另一部分贡献来自暴胀子衰变产生波洛尼粒子。该衰变道在运动学上是允许的，因为我们模型中暴胀子比波洛尼粒子更重。暴胀子衰变到一对波洛尼标量的微扰衰变率——例如综述可见文献 [41]——形式为

$$\Gamma_{\phi\to AA} = \frac{1}{192\pi}\frac{m_{\rm inf}^3}{M_{Pl}^2}. \tag{147}$$

One may expect that the non-perturbative preheating can be significantly increased due to a broad parametric resonance [84]. Inflaton and Polonyi fields are mixed as

可以预期非微扰预加热会因为宽参数共振得到显著增强 [84]。暴胀子场与波洛尼场混合如下:

$$\mathcal{L}_{\phi\phi\to AA} = \lambda\phi^2\bar{A}A \tag{148}$$

that follows from an expansion of the scalar potential with respect to both the scalar fields ϕ and A in (90) with the J -function (38). The broad parametric resonance could lead to an enhancement of the perturbative production of the Polonyi particles, up to the factor of $\mathcal{O}\left(10^5\right)$ - see, for example, Ref. [85] for numerical calculations of the broad resonance effects in supergravity. A calculation yields

这是对 (90) 式中标量势关于两个标量场 ϕ 和 A 、结合 (38) 式的 J 函数展开得到的结果。宽参数共振可以增强波洛尼粒子的微扰产生，增强因子最高可达 $\mathcal{O}\left(10^5\right)$ ——例如超引力中宽共振效应的数值计算可见文献 [85]。计算给出

$$\lambda = 10e^{7-2\sqrt{3}}\frac{\mu^2}{M_{Pl}^2}, \tag{149}$$

i.e., the coupling constant λ is of the order $O\left(10^{-7}\right)$ or less in PS supergravity. This small value fully compensates any possible enhancement of the Polonyi production by the broad parametric resonance. In other words, the Polonyi production from the inflaton decays is a perturbative process in the case of study. In summary, the grav-itino production from inflaton decays is sub-leading versus the Schwinger effect.

即，在普朗克星模型 (PS) 超引力中，耦合常数 λ 的量级为 $O\left(10^{-7}\right)$ 甚至更小。这个小值完全抵消了宽参数共振对波洛尼产生带来的所有可能增强。换句话说，在我们研究的情形中，暴胀子衰变产生波洛尼是微扰过程。综上，暴胀子衰变产生引力微子的贡献与施温格效应相比是次领头的。

Conclusion

结论

It was demonstrated in this chapter that the proposed extensions of the Starobinsky inflation model in supergravity can describe inflation in agreement with CMB measurements, as well as DM in the form of PBH and/or supermassive gravitino particles, all having the supergravitational origin, with a small number of free parameters. The viable inflationary models in supergravity can be based on either single-field or multi-field inflation. The power spectrum enhancement and the ultra-slow-roll phase, needed for PBH production, can be engineered via tachyonic instabilities of scalar fields in the double-inflation scenario; see also Ref. [86] for the similar approach in supergravity. Being the more fundamental theory of gravity (versus general relativity), supergravity has higher predictive power for phenomenology of the early universe. In turn, observational cosmology can be considered as a probe of supergravity and high-energy physics beyond the SM.

本章已证明，本文提出的超引力框架下斯塔罗宾斯基暴胀模型推广，可在仅有少量自由参数的前提下，描述符合宇宙微波背景测量结果的暴胀，以及原初黑洞 PBH 和/或超大质量引力微子形式的暗物质 DM，二者均起源于超引力。可行的超引力暴胀模型可基于单场或多场暴胀。PBH 形成所需的功率谱增强和极慢滚阶段，可通过双暴胀场景中标量场的快子不稳定性实现；关于超引力中的类似方法另见文献 [86]。作为比广义相对论更基础的引力理论，超引力对早期宇宙唯象学拥有更强的预言能力。反过来，观测宇宙学可作为探测标准模型 SM 之外的超引力与高能物理的探针。

The key theoretical tools also include manifest local supersymmetry provided by curved superspace and required for consistency of the phenomenological models, modified supergravity, specific mechanisms of spontaneous SUSY breaking, and the super-Higgs effect. It leads to high-scale SUSY and the gravitino mass of $O\left(10^{12}\right)$ GeV just below the inflationary scale. This value is consistent with the known value of Higgs mass and the two-loop calculations of the renormalization group equations relating the MSSM scale with the high scale of SUSY in the case of the gaugino coupling mixing parameter $\tan\beta$ close to one [87]. Having part of DM in the form of the massive LSP gravitinos produced at the end of inflation also implies that none of the SM superpartners was produced after inflationary reheating, which means no hope to detect sparticles in colliders.

关键理论工具还包括弯曲超空间提供的、唯象模型自洽性所要求的显式局域超对称，修正超引力，自发超对称破缺的特定机制，以及超希格斯效应。这些机制导致高标度超对称，引力微子质量为 $O\left(10^{12}\right)$ GeV，恰好低于暴胀能标。该值与已知希格斯质量一致，也符合在 gaugino 耦合混合参数 $\tan\beta$ 接近 1 时，联系最小超对称标准模型 MSSM 标度与高超对称标度的重整化群方程两圈计算结果 [87]。部分暗物质以暴胀末期产生的大质量最轻超对称粒子 LSP 引力微子形式存在，这还意味着暴胀重加热后没有产生任何 SM 超对称伙伴，因此不可能对撞机上探测到超对称粒子。

Supergravity can be embedded into compactified superstring/M-theory that gives its ultraviolet completion in quantum gravity. For instance, high-scale SUSY breaking, heavy particle DM, and connection to MSSM can be realized in the heterotic M-theory [88].

超引力可以嵌入紧致化超弦/M 理论，后者给出了超引力在量子引力中的紫外完备化。例如，高标度超对称破缺、重粒子暗物质以及与 MSSM 的联系都可以在杂化 M 理论中实现 [88]。

Significant part of DM may be in the form of PBH, with the PBH masses being in the range between 10^{17} g and 10^{21} g . The whole DM as the PBH is possible too, though after significant fine-tuning of the parameters. These PBH masses corresponding to the asteroid-size black holes are beyond the lower bound provided by Hawking radiation but are much lower the solar mass and the black hole masses discovered by the LIGO experiment [89].

暗物质的很大一部分可能以原初黑洞 PBH 的形式存在，PBH 质量范围在 10^{17} g 到 10^{21} g 之间。即使全部暗物质都由 PBH 构成也是可能的，只不过需要对参数进行大量微调。这些 PBH 对应小行星质量大小的黑洞，高于霍金辐射给出的下限，但远低于太阳质量，也远低于 LIGO 实验发现的黑洞质量 [89]。

PBH formation leads to GW because large scalar overdensities act as a source of stochastic GW background. Those GW may be detected by the future space-based GW interferometers such as LISA [70], TAIJI (old ALIA) [73], TianQin [72], and DECIGO [74]. The supergravity models also predict the GW stochastic background radiation that is sensitive to the inflationary parameters and the PBH mass spectrum. The NANOGrav Collaboration data [90] hints to the PBH as part of DM too [91].

PBH 形成会产生引力波 GW，因为大的标量过密度是随机引力波背景的源。这些引力波可被未来的天基引力波干涉仪探测到，例如 LISA[70]、太极 (原 ALIA)[73]、天琴 [72] 和 DECIGO[74]。超引力模型还预言了对暴胀参数和 PBH 质量谱敏感的随机引力波背景辐射。NANOGrav 合作组的数据 [90] 也支持 PBH 构成部分暗物质的观点 [91]。

Fine-tuning in the supergravity models amounts to fixing the parameter $M \sim 10^{-5} M_{\rm Pl}$ as the (Starobinsky) inflaton mass and the dimensionless parameter ζ needed for enough e-folds. The PBH masses found are compatible with all astrophysical and cosmological constraints [12].

超引力模型中的微调相当于固定参数 $M \sim 10^{-5} M_{\rm Pl}$ 作为 (斯塔罗宾斯基) 暴胀子质量，以及固定无量纲参数 ζ 来获得足够的暴胀 e-fold 数。得到的 PBH 质量符合所有天体物理与宇宙学约束 [12]。

The main takeaway from this chapter is that inflation, PBH formation, PBH/gravitino DM, and SUSY breaking can be unified in the supergravity framework and directly affect each other, leading to the intriguing unifying picture of inflation and DM, in which their parameter spaces are linked to each other. This scenario also suggests interesting phenomenology in the ultra-high-energy cosmic rays because superheavy Polonyi particles may also decay into the SM particles, as the secondaries, in top-bottom decays.

本章的核心结论是，暴胀、PBH 形成、PBH/引力微子暗物质和超对称破缺都可以统一在超引力框架中，且彼此直接关联，形成了暴胀与暗物质统一的有趣图景，二者的参数空间相互关联。该情景还预言了超高能宇宙线中有趣的唯象信号, 因为超重波洛尼粒子也会在自上而下衰变中衰变产生次级 SM 粒子。

Supergravity is often regarded as an extension of gravity at super-high-energy scales. With the whole DM being composed of superheavy LSP gravitinos, the only experimentally verifiable signature of SUSY would be just the DM. However, the new scalars of supergravity can play the active role during inflation, catalyze PBH formation, and produce GW radiation. The interactions of those scalars are dictated by local SUSY and are not assumed ad hoc, so that the supergravity models have the predictive power that can be falsified in future experiments. It gives us the reason to believe that indirect footprints of SUSY may be detected from GW physics rather than from high-energy particle colliders.

超引力通常被认为是超高能标下的引力扩展理论。如果全部暗物质都由超重 LSP 引力微子构成，那么超对称唯一可实验验证的信号就只有暗物质。然而，超引力的新标量可以在暴胀过程中发挥主动作用，催化 PBH 形成并产生引力波辐射。这些标量的相互作用由局域超对称决定，并非特设假设，因此超引力模型具备预言能力，可以被未来实验证伪。这让我们有理由相信，超对称的间接印记更可能从引力波物理而非高能粒子对撞机中被探测到。

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Cross-References

交叉引用

D-Branes

D 膜

- Moduli Stabilization in String Theory
  - 弦论中的模稳定
- Nonminimal Higgs Inflation and Initial Conditions in Cosmology
  - 非最小希格斯暴胀与宇宙学初始条件
- Quantum General Relativity and Effective Field Theory
  - 量子广义相对论与有效场论

Simple Supergravity

简单超引力

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